Math 160, Finite Mathematics for Business
Section 5.1 and 5.2 – Discussion Notes
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A **set** is a collection of things. The things in a set are its **elements**. We tend to use capital letters for sets. When describing a set we can explicitly list its elements or we can describe them. We use curly braces for this:

ex) A={1,4,6,7,8}, B={all first year students at UIC}, C={the 200 worst ideas of 2011}

The **empty set**, or **null set**, is a set containing no elements, denoted $\emptyset = \{\}$
A **subset** is a set that is contained in another set; specifically, we say A is a subset of B, denoted $A \subseteq B$ if every element of A is also an element of B.
If A and B are both subsets of each other, then we say the sets are equal.
If A is a subset of B and there is at least 1 element of B that is not in A, then we say A is a **proper subset** of B, denoted $A \subset B$.
The **universal set** U is the set containing all elements for the problem we are discussing.

Set Operations

Complement: The complement of a set A is the set of all elements in the universal set NOT contained in A, denoted $\overline{A}$. Sometimes the complement is denoted as A' or A^C.

ex) U={integers from 1 to 10} A={3,6,9}, $\overline{A}$={1,2,4,5,7,8,10} which are all elements from the universal set that are not found in A.

Union: The union of two sets A and B, denoted $A \cup B$ is the set of all elements that are found in A OR B (or both).

Intersection: The intersection of two sets A and B, denoted $A \cap B$, is the set of all elements found in both A AND B.

We define two sets to be "**disjoint**" if their intersection is the empty set (this means the two sets have no elements in common).

ex) U={a,b,c,d,e,f,g} R={a} S={a,b} and T={b,d,e,f,g}

a) $R \cup S$ ={a}∪{a,b} = {a,b}
b) $R \cap S$ ={a}∩{a,b} = {a} because that is the only element common to both
c) $\overline{T}$ = {a,c}, the only two elements of U that are not found in T
d) $\overline{T} \cup S$ ={a,c}∪{a,b}={a,b,c}, the three elements found in $\overline{T}$ or S

ex) U={1,2,3,4,5} R={1,3,5} S={3,4,5} T={2,4}

a) $R \cap S \cap T$ = {1,3,5}∩{3,4,5}∩{2,4}= {}= $\emptyset$ the empty set, because no elements are common to all three sets.
b) $R \cap S \cap \overline{T}$ = {1,3,5}∩{3,4,5}∩{1,3,5}={3,5}
c) $\overline{(S \cap T)}$ $= \overline{(\{3,4,5\} \cap \{2,4\})} = \overline{\{4\}} = \{1,2,3,5\}$
d) $\overline{S} \cup \overline{T}$ $= \overline{\{3,4,5\}} \cup \overline{\{2,4\}} = \{1,2\} \cup \{1,3,5\} = \{1,2,3,5\}$

This last example illustrates a property **called De Morgan's Law**:

For sets A and B, $\overline{(A\cup B)}=\overline{A}\cap\overline{B}$. Also $\overline{(A\cap B)}=\overline{A}\cup\overline{B}$

In words: the complement of a union is the intersection of complements. This shows how complement distributes over a union or intersection.

Logical Interpretation of Set Operations

We have the following interpretations of the set operations when translating English to set notation:

$A\cup B$ = "A and B" (also "A but B")

$A\cap B$ = "A or B"

$\overline{A}$ = "not A"

ex) U={all people}
C={people who like chocolate}, V={people who like vanilla}, S={people who like strawberry}
Translate to set notation:
a) {people who don't like strawberry}
This will be $\overline{S}$
b) {people who like vanilla or chocolate but not strawberry}
$(V\cup C)\cap\overline{S}$
c) {people who don't like any of the three flavors}
This can be states as $\overline{C}\cap\overline{V}\cap\overline{S}$ or $\overline{(C\cup V\cup S)}$. These equate to "not chocolate and not vanilla and not strawberry" and "not chocolate nor vanilla nor strawberry". You can see that these are equivalent, and in fact illustrate De Morgan's law for 3 sets.
d) {only like strawberry}
We could state this as "strawberry and not chocolate and not vanilla" or "strawberry and not chocolate nor vanilla". These sets would be $S\cap\overline{C}\cap\overline{V}$ and $S\cap\overline{(C\cup V)}$

Counting Elements in a Set

to count the elements in a set, say the number of elements in A, we use the notation n(A). This leads us to the complement rule:

If U is the universal set, then $n(\overline{A})=n(U)-n(A)$

General Addition Rule

$$n(A\cup B)=n(A)+n(B)-n(A\cap B) \text{ , and } n(A\cap B)=n(A)+n(B)-n(A\cup B)$$

A special case arises of A and B are disjoint, because then $n(A\cap B)=n(\emptyset)=0$
Addition Rule for Disjoint Sets: $n(A\cup B)=n(A)+n(B)$

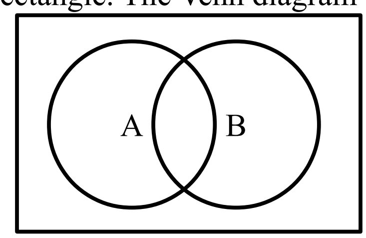
ex) From the 26 letters in English, 11 have vertical symmetry, 9 have horizontal symmetry and 4 have both types of symmetries. How many have neither?
n(U)=26 (all letters), n(V)=11, n(H)=9 and $n(V\cap H)=4$
we want to calculate $n(\overline{V\cup H})$.
By the complement rule, $n(\overline{V\cup H})=n(U)-n(V\cup H)=26-n(V\cup H)$.
By the general addition rule, $n(V\cup H)=n(V)+n(H)-n(V\cap H)=11+9-4=16$
So the number of letters with no symmetry is $n(\overline{V\cup H})=26-n(V\cup H)=26-16=10$ letters.

ex) 325 cars models have automatic transmission, 216 have power steering and 89 have both. How many models have at least 1 of the options?
Let A={cars with automatic transmission}, P={cars with power steering}.
n(A)=325, n(P)=216, n(A∩P)=89
We want to know $n(A\cup P)$ which is $n(A)+n(P)-n(A\cap P)=325+216-89=452$ by the General addition rule.

ex) Out of 100 investors, 80 own stocks and 70 own bonds. How many investors own both?
We assume that none of these 100 own neither stocks nor bonds – they could hardly be considered an investor in that case.
So $n(S\cup B)=100$, $n(S)=80$ and $n(B)=70$
We want to know $n(S\cap B)$ which is equal to $n(S)+n(B)-n(S\cup B)=80+70-100=50$ by the second form of the general addition rule.

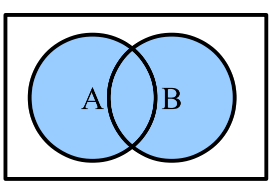
Venn Diagrams

A Venn diagram can be a useful way of depicting sets and set operations. We use circles to represent the sets, and enclose our diagram in a rectangle. The Venn diagram of A and B looks something like this:

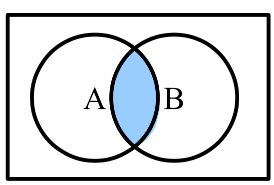


Here are some examples of set operations and their Venn Diagrams:

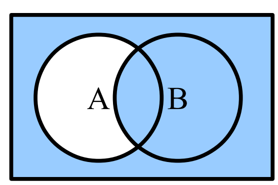
$A\cup B$



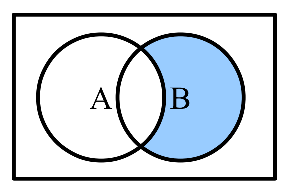
$A\cap B$



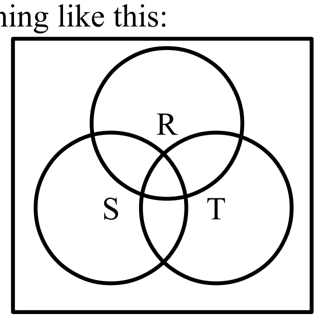
$\overline{A}\cup B$



$\overline{(A\cup \overline{B})}$

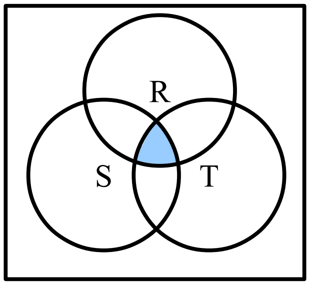


A three-set Venn diagram looks something like this:

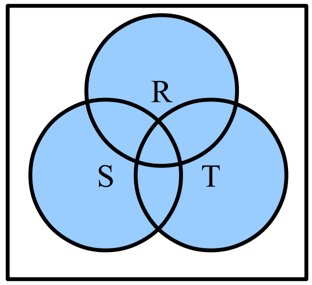


With some examples of set operations on 3 sets:

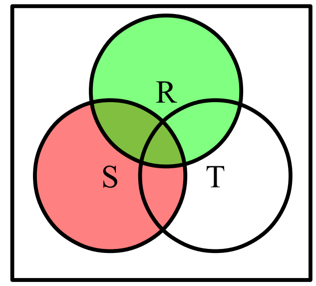
$R \cap S \cap T$



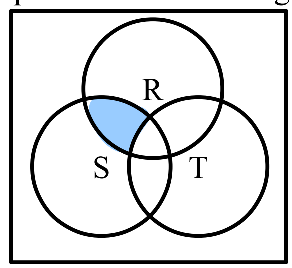
$R \cup S \cup T$



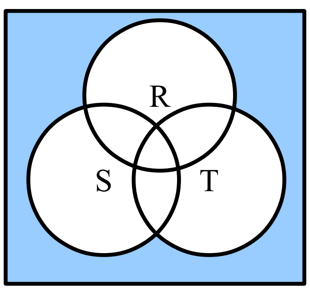
$R \cap S \cap \overline{T}$



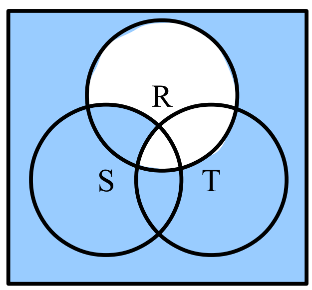
We look at the overlap of green, pink and the hatching:



$\overline{R} \cap \overline{S} \cap \overline{T}$



$\overline{R} \cup (S \cap \overline{T})$



$(R \cup \overline{S}) \cap (R \cup \overline{T})$