Math 160, Finite Mathematics for Business

Section 6.3: Assigning Probabilities - Discussion Notes
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To compare events we look at the probabilities of the events. **Relative frequency** and **experimental frequency** are measures derived from experimental results assigned to events. If we have a sample space S with n outcomes, $S = \{s_1, s_2, ..., s_n\}$ we associate probabilities $p_1, p_2, ..., p_n$ to each outcome such at $p_1 + p_2 + \cdots + p_n = 1$. This is a **probability distribution**.
A probability distribtution has the following properties:
1) Every outcome in the sample space is accounted for.
2) $p_1, p_2, ..., p_n$ are each between 0 and 1.
3) $p_1 + p_2 + \cdots + p_n = 1$
Furthermore, we now need to define an **elementary event**, which is simply an event that consists of a single outcome.

Addition Principle: If you have an event $E = \{s, t, u, ..., z\}$ then the Probability of E, written with notation $Pr(E) = Pr(s) + Pr(t) + Pr(u) + \cdots + Pr(z)$.
So the addition principle allows us to calculate the probability of an event by taking a sum of the probabilities of each outcome in the event. You may also note that $Pr(\emptyset) = 0$ and $Pr(S) = 1$ where S is the sample space.

Inclusion-Exclusion Principle: This is a direct application of the Inclusion-Exclusion Principle for counting the number of elements in the intersection of a set. The following are always true for events E and F:

$$Pr(E \cup F) = Pr(E) + Pr(F) - Pr(E \cap F)$$
$$Pr(E \cap F) = Pr(E) + Pr(F) - Pr(E \cup F)$$

Odds: Odds are related to probability but they are not the same. The odds of an event happening is a ratio comparing the probability of the event happening to the probability that it does not happen. This means we can convert odds to probability: Odds of E are "a to b" then $Pr(E) = \frac{a}{a+b}$.
We can convert a Probability to Odds as well. If Pr(E)=p, the odds are "p to (1-p)" but we must express these numbers are integers, so we may have to multiply them both by the lowest common denominator.

6.3.5) In Roulette you have a wheel with 38 slots numbered 1-36 (half red and half black) as well as green 0 and 00 slots. If the probability of any particular outcome is $\frac{1}{38}$, what is the probability of getting a green?
The event "Get a green" has two outcomes in it, each with the probability $\frac{1}{38}$, so the probability of this event is $\frac{1}{38} + \frac{1}{38} = \frac{2}{38} = \frac{1}{19}$.
What is the probability of getting a black?
There are 18 blacks in roulette, so the probability is $18 \cdot \frac{1}{38} = \frac{18}{38} = \frac{9}{19}$.

6.3.7) There is a horse race with three horses: Alfalfa, Basil and Cinnamon. If the probability that Alfalfa wins is $\frac{1}{2}$, and the probability that Basil wins is $\frac{1}{3}$, what is the probability that Cinnamon wins?
Because there are only 3 outcomes in the sample space and the probabilities of the three outcomes must add up to 1, and letting c represent the probability that Cinnamon wins, we can state $\frac{1}{2} + \frac{1}{3} + c = 1$. So $c = 1 - \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$.
What are the odds that Cinnamon wins?
The probability that Cinnamon wins is $\frac{1}{6}$, and the probability Cinnamon loses is $\frac{5}{6}$. We must represent the odds as integers, so we can multiply both by 6, the lowest common denominator. So the odds are 1 to 5 that Cinnamon wins.

6.3.25) Convert this table into a Probability Distribution:

# Colleges Applied To	**Probability**
1	**.17**
2 or fewer	**.29**
3 or fewer	**.43**
4 or fewer	**.59**
20 or fewer	**1**

This table is not a probability distribution because (1) the probabilities do not add up to 1, and (2) the events in column 1 are not mutually exclusive (i.e. there is overlap). In order to convert it into a probability distribution, we will need to break the data into non-overlapping outcomes. The outcomes can be defined as follows:

# Colleges Applied To	Probability
1	
2	
3	
4	
5 to 20	

Now we need to fill in the probabilities based on the original table. The first row can keep the probability of .17. The second row will have probability of .29-.17=.12. The Third row .43-.29, and so forth.

# Colleges Applied To	Probability
1	.17
2	.12
3	.14
4	.16
5 to 20	.41

What is the probability that a student applied to 3 or more colleges?
This event is the set of three outcomes: {3 schools, 4 schools, 5 to 20 schools}. This is found by taking the sum of the last 3 rows of our probability distribution table.. The sum is $.14 + .16 + .41 = .71$.

6.3.36) A basketball team has 8 players. What is the probability that at least two were born on the same day of the week?
There doesn't seem to be a lot of information in this problem, but if you think about it you will realize that there is enough to answer it. There are 7 days of the week and 8 basketball players. Is it even possible that no two of them were born on different days of the week? Even if the first 7 are born on Sunday, Monday, Tuesday, Wed, Thurs, Fri and Saturday, what day could the 8th player be born on that is different from all of the rest? It is certain that at least two are born on the same day of the week, and the probability of a certain event is 1.
This is an example of the **pidgeon hole principle**. This princple states that if there are n pigeon holes for pigeons to roost in and there are n+1 pigeons, you are certain that at least two of them must share a hole.