Math 160, Finite Mathematics for Business

Section 6.6: Tree Diagrams - Discussion Notes
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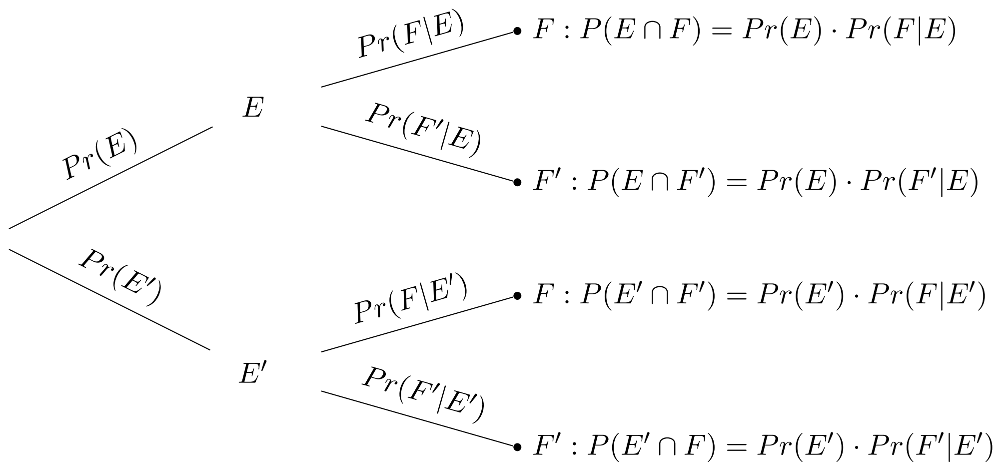
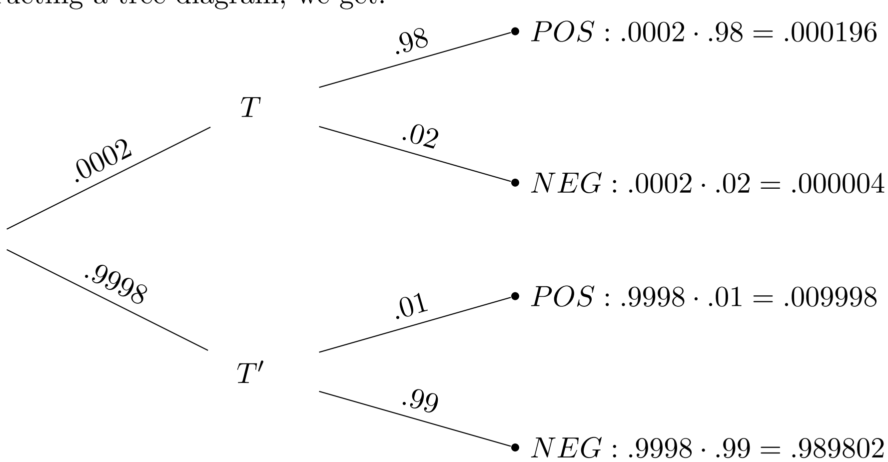
Tree diagrams are useful for illustrating conditional probabilities and getting the full picture. Let's consider a general example: Your first event is E and its complement is E', and the second event is F, and its complement is F'. We can depict a tree diagram as follows:



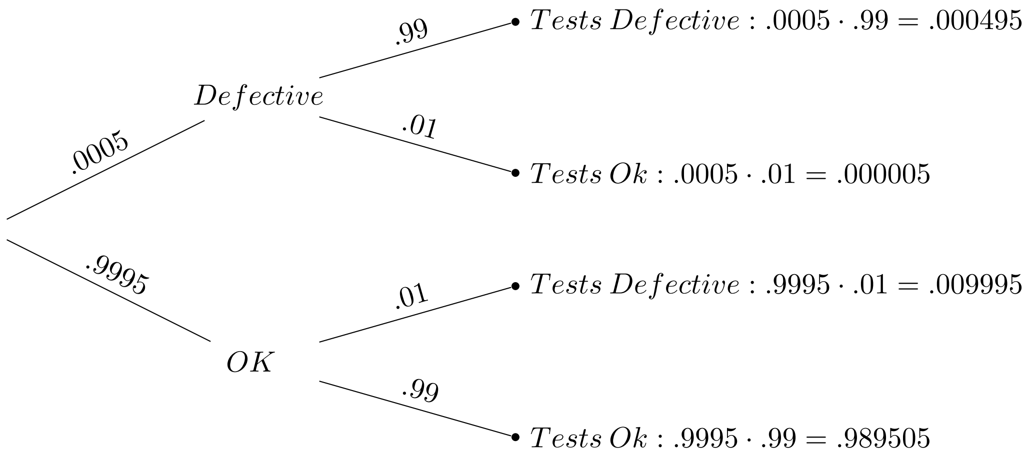
This can be extended to more than 2 events, or decisions with more than 2 possibilities. The first branch of the tree is the choice of E or E', with the lines labeled with the probabilities of E and E' respectively. Next each of these branches a second time based on F or F', and the branch is labeled with a conditional probability. Finally the end of each branch is given a probability which is the product of the probabilites leading to there. A more concrete example will help make it clearer.

ex) You have a population where TB occurs in .0002 of the population (.02%). A simple skin test is administered which gives the following results: If you are infected, the result is positive (POS) 98% of the time, and if you are healthy it gives a false POS 1% of the time. If someone tests POS, what is the probability that they have TB?
If we let event T:"Has TB", POS: "tests positive" and NEG: "Tests negative" then we have the following probabilities, given in the problem:
$Pr(T) = .0002$ and $Pr(T') = .9998$
$Pr(POS|T) = .98$, and it follows that $Pr(NEG|T) = 1 - .98 = .02$
$Pr(POS|T') = .01$ and similarly $Pr(NEG|T') = 1 - .01 = .99$
Constructing a tree diagram, we get:



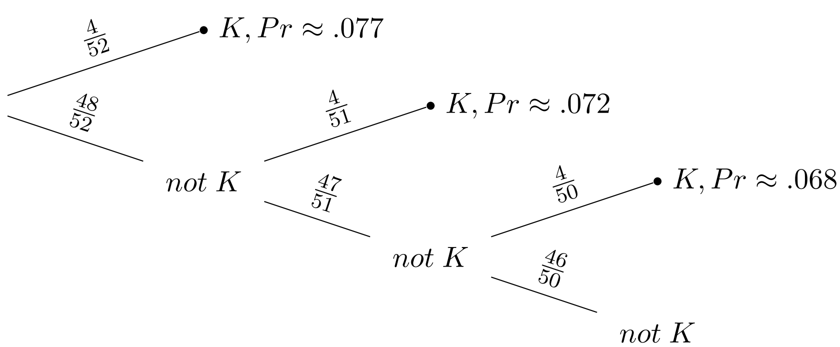
We want to calculate the probability someone has TB given that they tested positive, i.e. $Pr(T|POS)$. By the formula for conditional probability:
$Pr(T|POS) = \frac{Pr(T \cap POS)}{Pr(POS)} = \frac{.000196}{.000196+.009998} \approx .02$
It may be surprising that a positive test result only means one has a 2% chance of having TB, but you must consider that the incidence of TB is .02% to begin with - a positive test result raises your chances of having TB 100-fold. The reason that a positive test result isn't more conclusive is due to a) the low incidence of TB in the population combined with b) the 1% rate of false-positives.

6.6.36) A light bulb manufacturer produces lots of light bulbs, and .05% are defective. A light bulb tester is used in the production line and it is 99% effective (i.e. it will incorrectly say a working bulb is defective 1% of the time, and incorrectly say a defective bulb is working 1% of the time). If the bulb tester says the bulb is defective, what is the probability it is actually defective?
Filling in the tree diagram, we have:



$Pr(Defective|Tests\ Defective) = \frac{Pr(Def \cap Tests\ Def)}{Pr(Def)} = \frac{.000495}{.000495+.009995} \approx .0472$
So if a bulb tests defective, there is a 4.72% chance it is actually defective.

6.6.11) You draw cards from a normal deck of 52 cards until either a) you draw a King or b) You draw 5 cards. What is the probability you stop before the 4th draw?
Stopping before the 4th draw means pulling a King on the first, second or third draw. We can make a tree diagram for this.

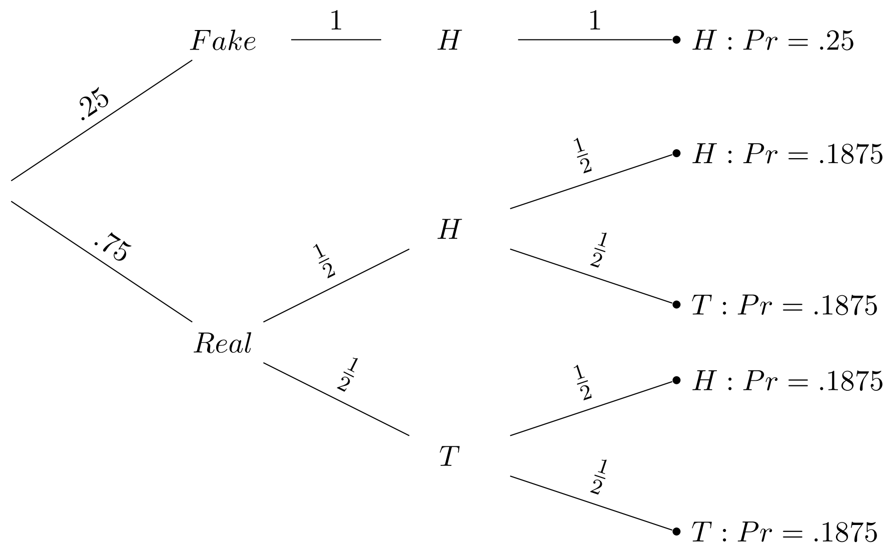


Note that in the first branch we have probabilities of $\frac{4}{52}$ and $\frac{48}{52}$ because there are 4 Kings in the deck, 48 non-kings and 52 cards total. On the second level of the tree there are only 51 cards left in the deck, so the probability of drawing a non-king will be $\frac{47}{51}$ because there is one fewer non-king in the deck, and so on. So the probability of drawing a king in one of the first three draws is $.077 + .072 + .068 = .217$

6.6.19) If you have three ordinary quarters and one fake quarter with head on both sides,

you put them in your pocket and randomly pick one and flip it twice - If you get Head on both flips, what is the probability that you have chosen the fake quarter?

First of all, the probability of drawing the fake quarter is $\frac{1}{4} = .25$ and the probability of drawing a normal one is .75. The tree diagram for this experiment looks like this:



What we want to calculate is the probability the coin is Fake given that you get HH, that is $Pr(Fake|HH) = \frac{Pr(Fake \cap HH)}{Pr(HH)} = \frac{.25}{.25 + .1875} \approx 0.57$