Fall 2014 Exam 2 Review TA: Brian Powers

1) a) $\frac{d}{dx}f(x) = \dfrac{\frac{1}{\ln 10 \cdot x}10^x - \log_{10}x \cdot 10^x \ln 10}{10^{2x}}$

b) $\frac{d}{dx}f(x) = -\frac{1}{\sqrt{1-25x^2}}(5)$

c) First:

$f(x) = \cos(h(x))$

let $h(x) = x^{\cos x}$

$\ln(h(x)) = \cos x \cdot \ln x$

$\frac{h'(x)}{h(x)} = -\sin x \cdot \ln x + \frac{\cos x}{x}$

$h'(x) = \left(\frac{\cos x}{x} - \sin x \ln x\right)x^{\cos x}$

$$\frac{d}{dx}\cos(x^{\cos x}) = -\sin(x^{\cos x})\cdot\left(\frac{\cos x}{x} - \sin x \ln x\right)x^{\cos x}$$

d) $\frac{d}{dx}f(x) = 3\tan^{-1}(x^3) + 3x\frac{1}{x^6+1}(3x^2)$

e) $\ln f(x) = x \ln(\sin x)$

$\frac{f'(x)}{f(x)} = \ln(\sin x) + \frac{x}{\sin x}\cdot\cos x \leftarrow \frac{\cos x}{\sin x} = \cot x$

$f'(x) = (\sin x)^x[\ln(\sin x) + x\cdot\cot x]$

2) $xy = 15$ so $y = \frac{15}{x}$

$3x + 5y = 3x + \frac{75}{x}$

Let $f(x) = 3x + \frac{75}{x}$

$f'(x) = 3 + -\frac{75}{x^2} = 0$ $\qquad f''(x) = +2\frac{75}{x^3}$

$3x^2 = 75$

$x^2 = 25$

$x = \pm 5$ only critical points. $\quad f''(5) > 0$ local min

$f''(-5) < 0$ local max

5,3 or −5,−3

(2 cont)

$3x + 5y$ has a local minimum at $x=5, y=3$
But if we are not restricted to positive numbers, then this can be unboundedly small (let $x \to -\infty$)
This can also be unboundedly large (let $x \to \infty$).

(3) a) Critical points when $f'(x)=0$. at $x=-3, 5$

b) Increasing on $(5,\infty)$ decreasing on $(-\infty,-3), (-3,5)$

c) $x=-3$ neither
$x=5$ local minimum

d) point of inflection $x=-3$ and $x=2.3$ (approx)
concave up on $(-\infty,-3)$ and $(2.3,\infty)$
down on $(-3, 2.3)$

(4) $g(x) = 9x^{1/3} - 4$

$g'(x) = \frac{1}{3}\cdot 9x^{-2/3} = 3x^{-2/3}$

a) $g''(x) = -\frac{2}{3}\cdot 3x^{-5/3} = -2x^{-5/3} = \dfrac{-2}{x^{5/3}}$

b) g is concave up when $x<0$ ~~But the domain is $x \geq 0$ so g is never concave up.~~

c) ~~Inflec There are no inflection points~~
Because $g(0)$ is defined, curve is continuous at $x=0$, then $x=0$ is an inflection point.

(5) a) domain of g is $x \geq -1$

b) $g'(x) = \sqrt{x+1} + \frac{1}{2}\frac{x}{\sqrt{x+1}}$

~~$\sqrt{x+1} + \frac{x}{2\sqrt{x+1}} = 0$~~

~~$2(x+1) + x\sqrt{x+1} = 0$~~

~~$2x+2 = -x\sqrt{x+1}$~~

~~$4x^2+8x+2 = x^3+x^2$~~

~~$0 = x^3-3x^2-8x-2$~~

~~check $x=1,-1,2,-2$ for zeros~~

~~$(-2)^3-3(-2)^2-8(-2)-2$~~

~~$= -8-12+16-2$~~

Note: $\sqrt{x+1} \geq 0$

$\frac{1}{2}\frac{x}{\sqrt{x+1}}$ is negative only for $-1 \leq x \leq 0$

~~5) $g(x) = \sqrt{x^3+x^2} = (x^3+x)^{\frac{1}{2}}$~~ ~~a) Domain is $x \geq -1$~~

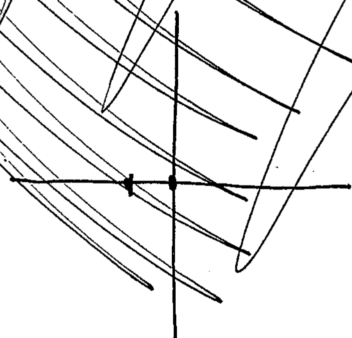
~~$g'(x) = \frac{1}{2}(x^3+x)^{-\frac{1}{2}}(3x^2+1) = \frac{3x^2+x}{2\sqrt{x^3+x^2}}$~~

~~$g'(x)=0$ only if $3x^2+1=0$, but this is impossible.~~

~~b) $g'(x)>0$ always, so $g(x)$ is never decreasing.~~

~~c) no local extrema exist, since $g'(x) \neq 0$ for any x.~~

~~d)~~



5) $g(x) = \sqrt{x^3+x^2} = (x^3+x^2)^{\frac{1}{2}}$

a) Domain $x \geq -1$

$g'(x) = \frac{1}{2}(x^3+x^2)^{-\frac{1}{2}}(3x^2+2x) = \frac{3x^2+2x}{2\sqrt{x^3+x^2}}$

$g'(x)=0 \rightarrow 3x^2+2x=0$

$x(3x+2)=0 \quad x=0$ or $x=\frac{-2}{3}$

~~But $x=0$ is an endpoint of the domain~~

derivative is undefined at $x=0$.

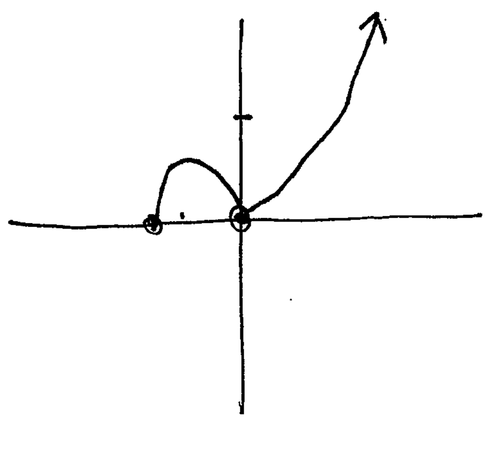
Denominator is always ≥ 0.
Numerator is negative on $(-\frac{2}{3}, 0)$

b) Intervals where $g(x)$ is decreasing $(-\frac{2}{3}, 0)$

c) Local extrema at $x=\frac{-2}{3}$, local max,

$x=0$ local min, cusp

d)

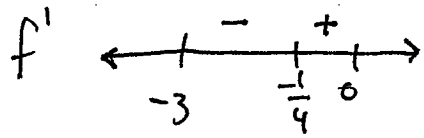


⑥ $f(x) = xe^{4x}$ on $[-3, 0]$

$f'(x) = e^{4x} + 4xe^{4x}$ $\qquad$ $1+4x > 0$ if $x > -\frac{1}{4}$

$e^{4x} + 4xe^{4x} = 0$ $\qquad$ $e^{4x} > 0$ always

$e^{4x}(1+4x) = 0$



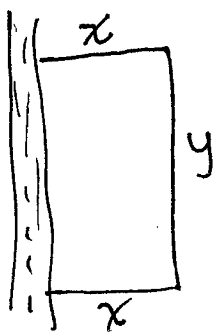
$x = -\frac{1}{4}$

$f(-3) = -3e^{-12}$ $\qquad$ $-\frac{1}{4}e^{-1} < -e^{-12} < 0$

$f(-\frac{1}{4}) = -\frac{1}{4}e^{-1}$ local min. and absolute min ☆

$f(0) = 0$ absolute max ☆

⑦



$xy = 20000 \rightarrow y = \frac{20000}{x}$

Minimize $2x + y = 2x + \frac{20000}{x}$

cost: $C(x) = 2x + \frac{20000}{x}$

$C'(x) = 2 - \frac{20000}{x^2}$

$2 - \frac{20000}{x^2} = 0$ if $x^2 = 10000$

ie $x = 100$ (-100 is not sensical)

Check $C''(x) = +2 \cdot \frac{20000}{x^3}$

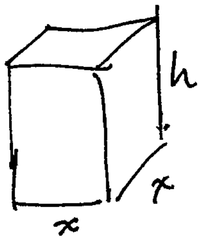
so $C''(100) > 0$

$x = 100$ is a local minimum.

$y = 200$

$\boxed{100 \times 200 \text{ ft minimizes cost}}$

⑧



$$\cancel{x^2h = 12} \qquad 12 = 2x^2 + 4hx$$

$$\cancel{h = \frac{12}{x^2}} \Rightarrow 4hx = 12 - 2x^2$$

$$h = \frac{12-2x^2}{4x}$$

a) $V = x^2\left(\frac{12-2x^2}{4x}\right) = \frac{12x}{4} - \frac{2x^3}{4} = 3x - \frac{1}{2}x^3$

b) $V' = 3 - \frac{3}{2}x^2$

$0 = 3 - \frac{3}{2}x^2 \Rightarrow x^2 = 2 \quad x = \sqrt{2}$

$V''(\sqrt{2}) = -3(\sqrt{2}) < 0$ so Local max.

$V(\sqrt{2}) = \cancel{2\left(\frac{12-4}{4\sqrt{2}}\right)} = 3\sqrt{2} - \frac{1}{2}2\sqrt{2} = 2\sqrt{2}$

Cube with side lengths $\sqrt{2}$

(Big surprise - a cube maximizes Volume if surface area is fixed).

⑨ $x\ln x + y\ln y = 1$

$\ln x + \frac{x}{x} + y'\ln y + \frac{y}{y}y' = 0$

$y'(\ln y + 1) = -1 - \ln x$

$y' = \frac{-1-\ln x}{1+\ln y}$

at $(e,1)$, $y' = \frac{-1-\ln e}{1+\ln 1} = \frac{-1-1}{1+0} = -2$

$y - 1 = -2(x - e)$ is the equation of the tangent line.

(10) Distance is $D(x) = \sqrt{(x-0)^2 + \left(\frac{x^2}{6}+4-13\right)^2}$ $\qquad \frac{18}{6}=3$

$= \sqrt{x^2 + \left(\frac{x^2}{6}-9\right)^2} = \sqrt{\underbrace{x^2 + \frac{x^4}{36} - 3x^2 + 81}}$

~~$D'(x) = \sqrt{\quad}$~~

Minimize $D(x)$ will be the same as minimizing this

$f(x) = \frac{x^4}{36} + \cancel{x^2} - 2x^2 + 81$

$f'(x) = \frac{4x^3}{36} - 4x = \cancel{\frac{x^3}{9} + 2x - 3}\ \frac{x^3}{9} - 4x$

~~$0 = x^3 + 18x - 27$~~

~~$x = 3?\quad 27 + 2 \cdot 27 - 27$~~

$0 = x^3 - 36x$

$0 = x(x^2 - 36)$

$x = \pm 6$ or $x = 0$

$D(0) = 9$

$D(6) = \sqrt{\frac{6^4}{36} - 2 \cdot 36 + 81} = \sqrt{81 - 36} = \sqrt{45}$

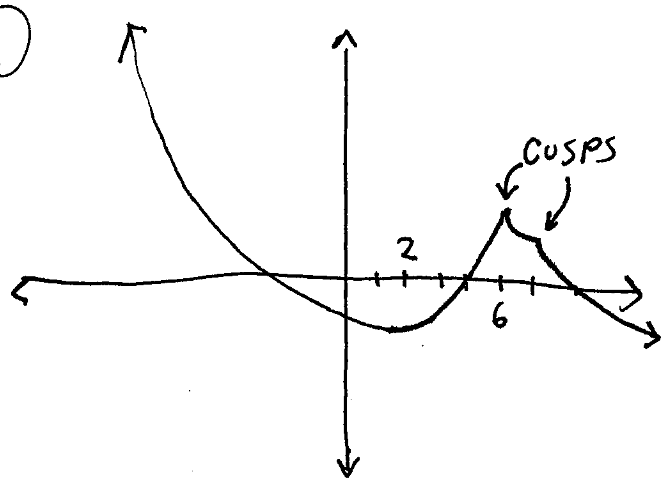
$D(-6) =$ same

as $x \to \infty$, Distance $\to \infty$ so $x = \pm 6$ minimizes distance. (Local & ABS minimum).

~~The point is~~ $y = \frac{6^2}{6} + 4 = 10$

The point(s) are $(6,10)$ and $(-6,10$

(11)



an example Graph.