Math 180: Calculus I **Fall 2014**

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We may use the following derivative rules now:

$$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\cos^{-1}x = -\frac{1}{\sqrt{1-x^2}}, \text{ for } -1 \le x \le 1$$
$$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2} \qquad \frac{d}{dx}\cot^{-1}x = -\frac{1}{1+x^2}$$
$$\frac{d}{dx}\sec^{-1}x = \frac{1}{|x|\sqrt{x^2-1}} \qquad \frac{d}{dx}\csc^{-1}x = -\frac{1}{|x|\sqrt{x^2-1}}, \text{ for } |x| > 1$$

Also to find the derivative of an inverse function $f^{-1}(y)$ at $y = y_0$, if $y_0 = f(x_0)$ and $f'(x_0) \neq 0$ then

$$(f^{-1})'(y_0) = \frac{1}{f'(x_0)}.$$

1. Evaluate the following derivatives:
   (a) $f(x) = \sin^{-1}(2x)$
   (b) $f(x) = \cos(\sin^{-1}(2x))$
   (c) $f(x) = \tan^{-1}(1/x)$
   (d) $f(x) = \csc^{-1}(\tan(e^x))$
   (e) $f(x) = 1/\tan^{-1}(x^2+4)$
2. Find the equation of the tangent line at the given point
   (a) $f(x) = \tan^{-1}(2x); (\frac{1}{2}, \frac{\pi}{4})$
   (b) $f(x) = \sec^{-1}(e^x); (\ln 2, \frac{\pi}{3})$
3. Find the derivative of $f^{-1}(y)$ at the given point.
   (a) $f(x) = 3x+4; (16, 4)$
   (b) $f(x) = x^2 - 2x - 3$ for $x \le 1; (12, -3)$
4. Use trig properties to prove the following identity. For what values of x is it true?
   $$\cos(2\sin^{-1}x) = 1 - 2x^2$$
   (Hint: $\cos 2\theta = 1 - 2\sin^2\theta$)
5. Consider $f(x) = \sin(2\sin^{-1}x)$.
   (a) What is the domain of f? Find the derivative $f'(x)$.
   (b) Find the equation of the tangent line to the graph when $x = \frac{1}{2}$.
   (c) Use $\sin 2\theta = 2\sin\theta\cos\theta$ and $1 = \cos^2\theta + \sin^2\theta$ to show $f(x) = 2x\sqrt{1-x^2}$.