Math 180: Calculus I   **Fall 2014**

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TA: Brian Powers

A few useful facts

$$\sum_{k=1}^{n} c = cn \qquad \sum_{k=1}^{n} k = \frac{n(n+1)}{2} \qquad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{k=1}^{n} k^3 \frac{n^2(n+1)^2}{4}$$

1. Approximate the displacement by using left-Riemann sums and the specified number of intervals.

$$v = \frac{1}{2t+1}, 0 \le t \le 8, n = 4$$

2. For the following functions, do the following:

   - Sketch the graph of the function on the interval
   - Calculate Δx and the grid points $x_0, x_1, ..., x_n$
   - Illustrate left and right Riemann sums, determine which sum underestimates the area under the curve.
   - Calculate both left and right Riemann sums.

   (a) $f(x) = x^2 - 1$ on $[2, 4], n = 4$

   (b) $f(x) = \cos x$ on $[0, \pi/2], n = 3$

3. Express the following in sigma notation

   (a) $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$

   (b) $3 + 8 + 13 + \cdots + 63$

   (c) $\frac{2}{3} - \frac{3}{4} + \frac{4}{5} - \frac{5}{6} + \cdots + \frac{100}{101}$

   (d) $\ln 2 + \sqrt{3} + \ln 4 + \sqrt{5} + \ln 6 \cdots + \ln 24 + \sqrt{25}$ (*Hint*: What values does $|\sin(k\pi/2)|$ take? Or what values does $\frac{1+(-1)^k}{2}$ take?)
   But this is still pretty ugly. We can't always make it pretty, but in this case we could if we changed our setup from the beginning. What if we instead group terms together, say the first term is $\ln 2 + \sqrt{3}$, the second term is $\ln 4 + \sqrt{5}$ and so on. Then we just need to be sure we're taking natural log of even numbers and square roots of teh odd numbers. We observe that $2k$ is even for any integer k, $2k + 1$ is odd. So we may write

   $$\sum_{k=1}^{1} 2\ln(2k) + \sqrt{2k+1}$$

4. Evaluate the following expressions, and also expand it out.

(a) $\sum_{k=1}^{10} k$

(b) $\sum_{k=1}^{6} (2k+1)$

(c) $\sum_{p=1}^{5} \left(2p + p^2\right)$
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