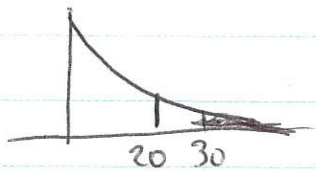
$\mu = \beta$

If $X \sim \exp(\beta)$ · same as $X \sim Gamma(\overset{\alpha}{1}, \beta)$



$f(x) = \frac{1}{\beta} e^{-x/\beta} \quad x \geq 0$

The time between equipment breakdowns at a factory follows an exp with mean 20 days.

What is the prob a month goes by without a breakdown?

$X \sim \exp(20)$ month = 30 days

$P(X > 30) = \int_{30}^{\infty} \frac{1}{20} e^{-x/20} dx \quad u = \frac{x}{20} \quad du = \frac{1}{20} dx$

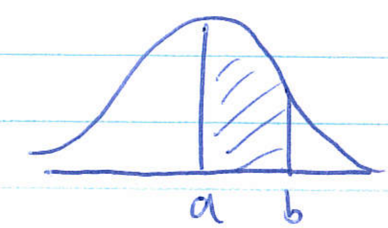
$= \int_{1.5}^{\infty} e^{-u} du = -e^{-u} \Big|_{1.5}^{\infty}$

$= \cancel{-e^{-\infty}} - -e^{-1.5} = e^{-1.5}$

$= .2231$

$F(x) = 1 - e^{-x/\beta}$

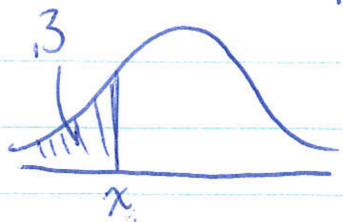
$1 - F(30)$



normalcdf(a, b, μ, σ)

$P(X > x) =$ normalcdf(x, ∞, μ, σ)

$1 -$ normalcdf$(-\infty, x, \mu, \sigma)$



$x = \mu + \sigma z$

For $X_1, \dots, X_n \overset{iid}{\sim} N(\mu, \sigma^2)$

$$E(\bar{X}) = \mu \quad Var(\bar{X}) = \frac{\sigma^2}{n}$$

$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

In General:

Thm Central Limit Theorem

If $X_1, \dots, X_n$ iid from a distribution with mean μ, and variance σ^2,

$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

follows a $N(0,1)$ distribution as $n \to \infty$

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even for $n \geq 30$ the distribution is approximately normal

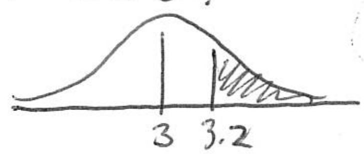
If distribution was "bellshaped" to begin with, n can be even smaller.

or $\bar{X} \underset{\sim}{\text{approx}} N\left(\mu, \frac{\sigma^2}{n}\right)$

If ~~weight~~ diameter of ballberrings follows a normal dist. with $\mu = 3mm$, $\sigma = .1mm$

What is the prob that a single ball berring is more than 3.2 mm wide?

$X \sim N(3, .1^2)$



$P(X > 3.2) =$

normal cdf(3.2, 10, 3, .1)

$\approx .02275$

Take a sample of 10 ballberrings, what is the prob that $\bar{X} > 3.2$?

$\frac{\sigma}{\sqrt{10}} = \sqrt{\frac{.1^2}{10}}$

$\bar{X} \sim N\left(3, \frac{.1^2}{10}\right)$ $\sigma_{\bar{X}} = .0316$

$P(\bar{X} > 3.2) =$ normalcdf(3.2, 10, 3, .0316)

$\approx 0.$

$\sigma^2_{\bar{X}} = \frac{\sigma^2}{n}$ $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$