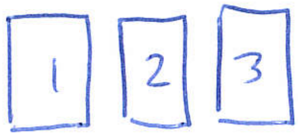
Monty Hall Game



Strategies Switch Not

$Pr\left(\substack{\text{1st Guess is} \\ \text{Correct}}\right) = \frac{1}{3}$

Pr (Behind one of 2 other Doors) $= \frac{2}{3}$

Let $\varepsilon > 0$

Sample Sum Distribution

$X_1, \dots, X_n$ iid ~~mean~~ from dist. with mean μ
Std. dev σ

$$S_n = \sum_{i=1}^{n} X_i \overset{\text{approx}}{\sim} N(n \cdot \mu,\ n\sigma^2)$$

Why $\text{Binom}(n, p) \longrightarrow \text{Normal}$ as $n \to \infty$

$X_i \sim \text{Bern}(p) \longleftarrow \mu = p,\ \sigma^2 = pq$

$S_n = Y = X_1 + \cdots + X_n \sim \text{Bin}(n, p)$

if n large, Y is approx Normal mean $= n\mu = np$

var $n\sigma^2 = npq$

Facts about Chi-Square Dist.

$\chi^2(v)$ $v =$ the "degrees of freedom"

is same as Gamma with $\alpha = \frac{v}{2}$, $\beta = 2$

mean of $\chi^2(v)$ is v,

var of $\chi^2(v)$ is $2v$

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if $Z \sim N(0,1)$

$Z^2 \sim \chi^2(1)$

if $Z_1, Z_2, \ldots, Z_n$ iid $N(0,1)$

$\sum_{i=1}^{n} Z_i^2 \sim \chi^2(n)$

$$S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2 \qquad (n-1)(S^2) = \sum_{i=1}^{n} (X_i - \bar{X})^2$$

add $\bar{X} - \bar{X}$

$$\sum_{i=1}^{n} (X_i - \mu)^2 = \sum_{i=1}^{n} \left((X_i - \bar{X}) + (\bar{X} - \mu)\right)^2$$

$$= \sum_{i=1}^{n} (x_i - \bar{x})^2 + \sum_{i=1}^{n} (\bar{X} - \mu)^2 + 2 \sum_{i=1}^{n} (x_i - \bar{x})(\bar{X} - \mu)$$

$$= \boxed{\sum_{i=1}^{n} (x_i - \bar{x})^2 + n(\bar{X} - \mu)^2} + \cancel{2(\bar{X} - \mu)\left(\sum_{i=1}^{n} (x_i - \bar{x})\right)}^{=0}$$

$\sum x_i - n\bar{x}$

$\bar{X} = \frac{\sum x_i}{n}$

$n\bar{x} = \sum x_i$

$$\frac{1}{\sigma^2} \sum_{i=1}^{n} (X_i - \mu)^2 = \frac{(n-1)S^2}{\sigma^2} + \frac{(\bar{X} - \mu)^2}{\sigma^2/n}$$

$\sum_{i=1}^{n} \left(\frac{X_i - \mu}{\sigma}\right)^2 \sim \chi^2(n)$

$\frac{(n-1)S^2}{\sigma^2}$: $\chi^2(n-1)$

$\frac{(\bar{X} - \mu)^2}{\sigma^2/n}$: $\chi^2(1)$

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1) \quad \text{if } X_1, \dots, X_n \text{ iid } N(\mu, \sigma^2)$$

even if X's are not normal, this approx. true as $n \to \infty$

(ex) Car batteries lifetimes are normally dist. with supposed std. dev. of 1 year. 5 batteries tested, we get 1.9, 2.4, 3, 3.5, 4.2 years.

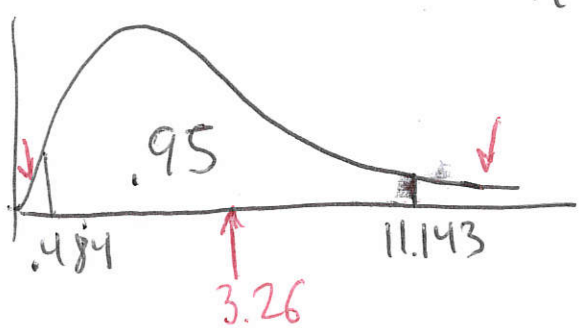
Should we suspect that σ has changed?

$\bar{x} = 3$

$S^2 =$ $\quad s^2 = .90277^2$

If $\sigma = 1$ then $\frac{(4)S^2}{1} \sim \chi^2(4)$

$4(.90277)^2 = 3.26$



CLT

$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

$$S_n \sim N(n\mu, n\sigma^2)$$

$$\bar{X}_1 - \bar{X}_2 \sim N\left(\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma^2}{n_2}\right)$$