Gamma Distribution

def The <u>gamma function</u> $\Gamma(\alpha)$

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} \cdot e^{-x}\, dx \qquad \alpha > 0$$

- Recursive in that $\Gamma(\alpha) = (\alpha-1)\Gamma(\alpha-1)$
- If $\alpha \in \mathbb{Z}^+$, $\Gamma(\alpha) = (\alpha-1)!$

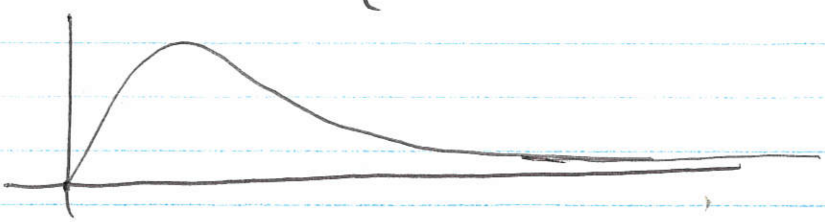
$$\Gamma(5) = 4!$$

- $\Gamma(\frac{1}{2}) = \sqrt{\pi}$ $\qquad \int_0^\infty \frac{e^{-x}}{\sqrt{x}}\, dx = \sqrt{\pi}$

If we have a Poisson Process where the average # occurrences per unit time is λ, If we want to measure the length of time until α occurrences, this follows a <u>Gamma Distribution</u> with parameters α, $\beta = \frac{1}{\lambda}$

If $X \sim \text{Gamma}(\alpha, \beta)$

$$f(x) = \begin{cases} \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-\frac{x}{\beta}} & x > 0 \\ 0 & x \leq 0 \end{cases}$$



If $X \sim \text{Gamma}(\alpha, \beta)$

$E(X) = \alpha\beta \qquad Var(X) = \alpha\beta^2$

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x}\, dx$$

Incomplete Gamma function (Lower, regularized)

$$F(x;\alpha) = \int_0^x \frac{y^{\alpha-1} e^{-y}}{\Gamma(\alpha)}\, dy$$

$$= 1 - \text{poissoncdf}(x, \alpha - 1)$$

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