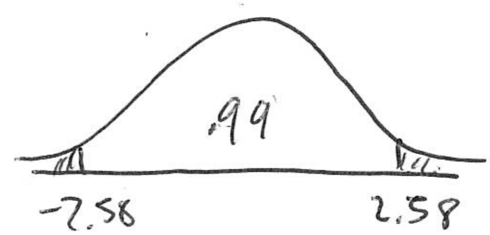
Stat 381 4/8/2015

ex Sampled 45 transistors and measured the voltage transmitted.
assuming σ is known, $\sigma = .2$ volts
we found $\bar{x} = 40$ v.
Construct a 99% confidence interval for μ

$100(1-\alpha)\%$ CI is $\bar{x} \pm z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}$

$\alpha = .01$

$z_{.005} = 2.58$



$$40 \pm 2.58\left(\frac{.2}{\sqrt{45}}\right)$$

$$(39.923, 40.077)$$

We may want to fix a margin of error.
say I want to estimate μ $\pm .01$ volts
we can find n which gives the desired sized interval.

$$ME = z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}} \leq \varepsilon$$

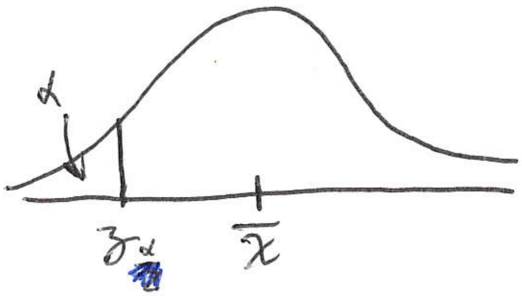
$$\frac{z_{\frac{\alpha}{2}}\sigma}{\varepsilon} \leq \sqrt{n}$$

$$\left(\frac{z_{\frac{\alpha}{2}} \cdot \sigma}{\varepsilon}\right)^2 \leq n$$

$$\left(\frac{2.58 \cdot .2}{.01}\right)^2 = 2662.56$$

need 2663 samples

One-sided confidence bounds.



ex Car batteries

sample 36 batteries

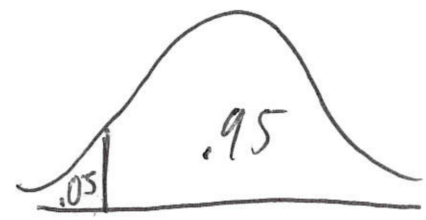
$\bar{x} = 3$ yrs

known $\sigma = .5$ yrs

one-sided bound

$$\bar{x} - z_\alpha \cdot \frac{\sigma}{\sqrt{n}}$$

95% Confidence bound



invNorm(.05) = -1.64

$$3 - 1.64\left(\frac{.5}{\sqrt{36}}\right) = 2.863$$

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if σ is unknown we use S as an estimate.

but in this case $\frac{\bar{x} - \mu}{S}$ follows a T-distr. with $n-1$ degrees of freedom

100(1-α)% CI in this case is

$$\bar{x} \pm t_{\frac{\alpha}{2}} \cdot \frac{S}{\sqrt{n}}$$

~~Say test # attend~~

estimate average attendance at City Hall meetings.

sample 7 random meetings,

get $\bar{x} = 52.7$, $s = 12$

a 90% confidence interval is

$$52.7 \pm \boxed{t_{\frac{\alpha}{2}} \cdot \frac{12}{\sqrt{7}}} \leftarrow \text{margin of error}$$

$\frac{\alpha}{2}$ $n-1$ $t_{\frac{\alpha}{2}} = 1.943$ 6 d.f.

invT(.05, df)

$\frac{12}{\sqrt{7}} \leftarrow$ standard error of $\bar{x}$

Comparing 2 populations

eg determine if ~~avg~~ students are happier in the class of 20~~0~~5 or 2016

(seniors) (juniors)

known

Sample 70 juniors $\bar{x}_j = 7.2$ $\sigma_j = 3$

85 seniors $\bar{x}_s = 8.4$ $\sigma_s = 2.4$

95% estimate an interval for $\mu_j - \mu_s$

CI

$(z_{.025} = 1.96)$ $\bar{x}_j - \bar{x}_s \pm 1.96\sqrt{\frac{3^2}{70} + \frac{2.4^2}{85}}$

$-1.2 \pm .8684$

$(-2.0684, -.3316)$

remember $\bar{X}_1 - \bar{X}_2 \sim N\left(\mu = \mu_1 - \mu_2,\ \sigma = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right)$

$\hookrightarrow E(\bar{X}_1 - \bar{X}_2) = \mu_1 - \mu_2$

$S.E.(\bar{X}_1 - \bar{X}_2) = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

$100(1-\alpha)\%$ CI for $\mu_1 - \mu_2$ [as long as $n_1, n_2 > 30$, σ_1, σ_2 known]

$$\bar{x}_1 - \bar{x}_2 \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

If make $100(1-\alpha)\%$ C.I for $\mu_1 - \mu_2$

and σ_1, σ_2 unknown <u>but</u> assumed $\sigma_1 = \sigma_2 = \sigma$

in this case $\bar{x}_1 - \bar{x}_2 \pm t_{\frac{\alpha}{2}} \cdot SE(\bar{x}_1 - \bar{x}_2)$

2 samples of sizes n_1 and n_2 respectively.

- degrees of freedom $n_1 + n_2 - 2$

$SE(\bar{x}_1 - \bar{x}_2) = \sqrt{\frac{\sigma^2}{n_1} + \frac{\sigma^2}{n_2}} = \sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$

use estimate of σ

sampling gives us S_1, S_2

pooled sample variance

- $S^2_{pooled} = \dfrac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1 + n_2 - 2}$

$SE(\bar{x}_1 - \bar{x}_2) = S_{pooled}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$