

## 1 Memorize

If  $X \sim \text{Bern}(p)$ ,  $E(X) = p$ ,  $\text{Var}(X) = pq$ .

If  $X \sim \text{Binom}(n, p)$ ,  $f(x) = \binom{n}{x} p^x q^{n-x}$ ,  $E(X) = np$ ,  $\text{Var}(X) = npq$ .

If  $X \sim \text{Poisson}(\lambda t)$ ,  $E(X) = \text{Var}(X) = \lambda t$ .

If  $X_i$ s are independent,  $E(\bar{X}) = \mu$ ,  $\text{Var}(\bar{X}) = \sigma^2/n$  regardless of the distribution.

CLT says:  $\bar{X} \sim N(\mu, \sigma^2/n)$  approximately, or  $(\bar{X} - \mu)/(\sigma/\sqrt{n}) \sim N(0, 1)$

If  $X \sim \text{Unif}_C(a, b)$ ,  $f(x) = \frac{1}{b-a}$  for  $x \in [a, b]$ ,  $E(X) = \frac{a+b}{2}$ ,  $\text{Var}(X) = \frac{(b-a)^2}{12}$ .

## 2 Don't Have to Memorize

Poisson pmf:  $f(x) = \frac{(\lambda t)^x e^{-\lambda t}}{x!}$  for  $x \in \mathbb{N}$ .

Hypergeometric, Geometric, Negative binomial pmf.

Normal distribution pdf

Gamma distribution pdf

t-distribution pdf

## 3 Review Problems

1. Give the name of the distribution best used to model each random variable:
  - (a) Whether or not the first life savers in the package is pineapple
  - (b) The number of cherry starburst candies in a package of 12
  - (c) The number of mystery dum-dums you need to unwrap until you get a root beer
  - (d) The number of trick-or-treaters until you've seen 10 Elsas (from Frozen)
  - (e) The number of red candies in a handful of 10 drawn from a bowl of red, blue and yellow candies
  - (f) The number of trick-or-treaters coming to your door between 8:05 and 8:10pm
  - (g) The length of time between trick-or-treaters
  - (h) The length of time until 10 trick-or-treaters in total have arrived
  - (i) The height of the next trick-or-treater
  - (j) The angle between the seconds hand and minutes hand when the first trick-or-treater arrives.

2. In a game, you are allowed to roll a pair of fair dice 24 times. If you get a double six (i.e. you roll (6,6)) at least 4 times you win. What is the probability you win this game?
  
3. Let  $X$  be the number of customer logins on a website during an hour. Assume  $X$  has a Poisson distribution with a mean of 120 login requests per hour.
  - (a) What is the probability that no one requests to log on the site in the next ten minutes?
  - (b) Let  $W$  be the time in minutes between the 2nd and 3rd request. What is the distribution name of  $W$ ? What is the expected value of  $W$ ?
  - (c) Let  $T$  be the time in minutes until the 4th request. What is the distribution name of  $T$ ? What is the expected value of  $T$ ?
  
4. At a certain gas station, the number of lottery winners each month follows a Poisson distribution with mean 1.
  - (a) What is the probability that there are no more than 3 winners this year?
  - (b) What is the probability there is at least one winner during a month?
  - (c) What is the probability that there is at least one winner every month during the year?
  
5. Let  $X_1, X_2, \dots, X_{36}$  be a random sample from a continuous Uniform distribution over  $[0, 2]$ .
  - (a) Find the pdf for the population.
  - (b) Find the mean and variance for the population.
  - (c) Find the mean and variance for the sample mean  $\bar{X} = (X_1 + \dots + X_{36})/36$ .
  - (d) Use the Central Limit Theorem to estimate  $P(0.9 < \bar{X} < 1.1)$ .
  
6. If a r.v.  $X$  follows a normal distribution with mean 80 and standard deviation 20, find the following using the Empirical rule (68 - 95 - 99.7 rule).
  - (a)  $P(X \geq 20)$
  - (b)  $P(|X - 80| \leq 40)$
  - (c)  $P(X \leq 100)$
  - (d)  $P(X > 120)$
  
7. The size of a download from a server follows a normal distribution with mean 4 MB and standard deviation 0.8 MB.
  - (a) What is the probability that a download exceeds 5 MB?

- (b) What are the 25th and 75th percentiles?
  - (c) If someone downloads 5 files, what is the probability that the total download is more than 22 MB?
8. Statistics show that on an average weekend night 1 out of 10 drivers on the road is drunk.
- (a) If 20 drivers are checked, what is the expected number of drunk drivers?
  - (b) If 20 drivers are checked, what is the probability that at least 1 of them is drunk?
  - (c) If a sample of 200 drivers are checked, what is the mean and variance for the number of drunk drivers?