Exam 2 STAT 381, Applied Statistical Methods I, Spring 2015

Name:

For full credit you must show your work. If you use a calculator function, please write down the calculator input you used to get your answer. Read each problem carefully.

1. You make a bet for \$100: If you can make at least 7 out of 10 baskets, you win. Suppose your probability of making each basket is 3/4, and each attempt is independent.

   (a) (10 points) If X is the number of baskets you make, what is the distribution of X (give the distribution name and parameter values).
   Binomial Distribution, $n = 10$, $p = 3/4$.

   (b) (10 points) What is the probability you win the bet?
   $P(Win) = P(X \geq 7) = 1 - P(X \leq 6) = 1 - \texttt{binomcdf(10,.75,6)} = .7759$

   (c) (Bonus 5 points) What is the expected value for this bet?
   $P(Win) = .7759$, so $P(Lose) = 1 - .7759 = .2241$. You gain \$100 if you win, lose \$100 if you lose, so the expected winnings is $100(.7759) - 100(.2241) =$ \$55.18

2. At work, the number of phone calls each hour follows a Poisson distribution with mean of 5.

   (a) (10 points) What is the probability that 30 minutes goes by without the phone ringing?
   The average number of calls in 30 minutes is 2.5, and the number of calls in 30 minutes follows a Poisson distribution as well. Let Y be the number of calls in 30 minutes. $P(Y = 0) = \texttt{poissonpdf(2.5,0)} = e^{-2.5} = .082$

   (b) (10 points) Let T be the total number of phone calls over a 40 hour work week. What is $E(T)$ and what precise distribution does it follow (do not use Central Limit Theorem)?
   The number of calls in any length of time should follow a Poisson distribution, the mean will be proportional to the length of time. So $E(T) = 40(5) = 200$, and T follows a Poisson distribution (with $\lambda = 200$)

   (c) (10 points) Let $H = \frac{T}{40}$, that is, the average hourly number of phone calls during this week. What distribution does H follow (approximately) under the Central Limit Theorem (Give the distribution name, μ_H and σ_H).
   H is simply a sample average from 40 independent hours. If the number of calls in each hour are $X_1, \ldots, X_{40}$, with $X \sim \text{Poisson}(5)$, then $\mu_X = 5, \sigma_X = \sqrt{5}$. So $H = \bar{X}$ is approximately Normal with mean $\mu_H = 5$, and $\sigma_H = \sqrt{5}/\sqrt{40} = .3536$.

   (d) (10 points) What is the approximate probability that $H > 5.5$, by Central Limit Theorem?
   $P(H > 5.5) \approx \texttt{normalcdf(5.5,1000,5,.3536)} = .0787$

(e) (Bonus 5 points) What is the exact probability that $H > 5.5$? *(Hint: you have to use the distribution of T)*
$P(H > 5.5) = P(40H > 40{\cdot}5.5) = P(T > 220) = 1-$`poissoncdf(200,220)` $=$ .0753, Which means the approximation isn't too far off.

3. Let X follow a Normal distribution with $\mu = 50$ and $\sigma = 15$.

   (a) (10 points) Find $P(44 < X < 58)$.
   `normalcdf(44,58,50,15)` $= .3585$

   (b) (10 points) Find x such that $P(X > x) = .01$ (i.e. the 99th percentile).
   $x =$ `invNorm(.99,50,15)` $= 84.895$

   (c) (10 points) If $X_1, \ldots, X_9$ are independently drawn from this population, with $\bar{X} = \frac{1}{9}(X_1 + \cdots + X_9)$, find $P(44 < \bar{X} < 58)$.
   $\bar{X}$ follows a Normal distribution with $\mu_{\bar{X}} = 50$, $\sigma_{\bar{X}} = 15/\sqrt{9} = 5$. So $P(44 < \bar{X} < 58) =$ `normalcdf(44,58,50,5)` $= .8301$

4. (10 points) The length of time between births at a hospital follows an Exponential distribution with a mean of 6 hours. If a baby is born at 9am, what is the probability another baby is born before 10am?
Method 1: Let X be the length of time in hours until the next birth, which follows an exponential distribution with $\beta = 6$.

$$P(X < 1) = \int_0^1 \frac{1}{6} e^{-x/6} dx = -e^{-x/6}\Big|_0^1 = -e^{-1/6} - -1 = .1535$$

Method 2: The number of babies born in 1 hour is on average 1/6, so Y, the number of babies born in the next hour follows a Poisson distribution with $\lambda = 1/6$.

$$P(Y \geq 1) = 1 - P(Y = 0) = 1 - e^{-1/6} = .1535$$

Some Distributions

Binomial(n, p)

$$f(x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, 2, \ldots, n$$

Geometric(p)

$$f(x) = p(1-p)^{x-1}, q \quad x = 1, 2, \ldots$$

Negative Binomial(k, p)

$$f(x) = \binom{x-1}{k-1} p^k (1-p)^{x-k}, \quad x = k, k+1, \ldots$$

Poisson(λt)

$$f(x) = \frac{(\lambda t)^x e^{-\lambda t}}{x!}, \quad x = 0, 1, 2, \ldots$$

Normal(μ, σ^2)

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}, \quad x \in \mathbb{R}$$

Gamma(α, β)

$$f(x) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x/\beta}, \quad x > 0$$

Exponential(β)

$$f(x) = \frac{1}{\beta} e^{-x/\beta}, \quad x > 0$$

Chi-Squared(v)

$$f(x) = \frac{1}{\Gamma(\frac{v}{2}) 2^{v/2}} x^{v/2-1} e^{-2x}, \quad x > 0$$