

MATH 417 HOMEWORK 4

This homework is due Wednesday September 24 in the beginning of class. You may collaborate on the homework. However, the final write-up must be yours and should reflect your own understanding of the problem. Please be sure to properly cite any help you get.

Problem 1 Prove the following formulae involving trigonometric functions

- (1) $2 \sin(z_1) \cos(z_2) = \sin(z_1 + z_2) + \sin(z_1 - z_2)$
- (2) $\cos(z_1 + z_2) = \cos(z_1) \cos(z_2) - \sin(z_1) \sin(z_2)$
- (3) $|\cos(z)|^2 = \cos^2(x) + \sinh^2(y)$, where $z = x + iy$.

Problem 2 Show that

- (1) $|\sinh(y)| \leq |\sin(z)| \leq \cosh(y)$; and
- (2) $|\sinh(y)| \leq |\cos(z)| \leq \cosh(y)$, where $z = x + iy$.

Problem 3 Calculate $\text{Log}(1 + i)$ and $\text{Log}(-ei)$. Calculate $\log(e^2)$ and $\log(1 - i)$.

Problem 4 Find the principal values of i^{2i} and $(1 + i)^{4i}$. Find all values of 2^i .

Problem 5 Find all the values of $(1 + \sqrt{3}i)^{3/2}$.

Extra Credit: Find the genus of the following Riemann surfaces

- (1) $y^2 = x(x^2 - 1)(x^2 - 4)$
- (2) $y^2 = (x^2 - 1)(x^2 - 4)(x^2 - 9)(x^2 - 16)$