MATH 417 HOMEWORK 9

This homework is due Friday November 7 in the beginning of class. You may collaborate on the homework. However, the final write-up must be yours and should reflect your own understanding of the problem. Please be sure to properly cite any help you get.

Problem 1 For each of the following functions find all the isolated singularities. Determine whether they are removable singularities, poles or essential singularities. If the function has a pole, determine the order of the pole.

$$(a) \ \frac{\sin(z^2)}{z^2} \quad (b) \ \frac{\cos(z) - 1}{z^4} \quad (c) \ (\tan z)^2 \quad (d) \ \tan(z^2)$$

Problem 2 Find the residues of $f(z)dz$ at all the isolated singular points in the complex plane for the following functions $f(z)$

$$(a) \ \frac{z + 5}{z(z^2 + 1)} \quad (b) \ \frac{2z + 1}{(z - 1)^2(z + 1)} \quad (c) \ z^2 e^{1/z}$$

Problem 3 Let C be the positively oriented simple closed contour $|z| = 3$. Calculate the following integrals.

$$(a) \ \int_C \frac{ze^z}{z^2 - 1}dz \quad (b) \ \int_C \frac{\cosh(\pi z)}{z(z^2 + 1)}dz \quad (c) \ \int_C \frac{z^3 e^{1/z}}{1 + z^3}dz$$

Problem 4 For the following two problems you must justify your answer.
  (1) Find all functions $f(z)$ that are analytic in the entire complex plane and satisfy $2|\sin(z)| \geq |f(z)|$.

  (2) Find all functions $f(z)$ that are analytic in the entire complex plane and satisfy $2|f(z)| \geq |\sin(z)|$.

Problem 5 For each of the following functions find the order of zero at $z = 0$.

$$(a) \ \cos(z^2) - 1 \quad (b) \ \sin(z^3) \quad (c) \ (e^z - 1)^3 \quad (d) \ z^7 - 12z^5 + z^3$$