Math 180, Exam 1, Fall 2009
Problem 1 Solution

1. Evaluate the following limits.

(a) $\lim_{x\to 2} \dfrac{x^2 - 6x + 8}{x - 2}$

(b) $\lim_{x\to 1} \dfrac{x^2 - 6x + 8}{x - 2}$

Solution:

(a) Upon substituting $x = 2$ into the function $f(x) = \dfrac{x^2 - 6x + 8}{x - 2}$ we find that

$$\frac{x^2 - 6x + 8}{x - 2} = \frac{2^2 - 6(2) + 8}{2 - 2} = \frac{0}{0}$$

which is indeterminate. We can resolve the indeterminacy by factoring.

$$\begin{aligned} \lim_{x\to 2} \frac{x^2 - 6x + 8}{x - 2} &= \lim_{x\to 2} \frac{(x-2)(x-4)}{x-2} \\ &= \lim_{x\to 2} (x - 4) \\ &= 2 - 4 \\ &= \boxed{-2} \end{aligned}$$

(b) The function $f(x) = \dfrac{x^2 - 6x + 8}{x - 2}$ is continuous at $x = 1$. In fact, it is continuous at all $x \neq 2$. Therefore, we can evaluate the limit using substitution.

$$\lim_{x\to 1} \frac{x^2 - 6x + 8}{x - 2} = \frac{1^2 - 6(1) + 8}{1 - 2} = \boxed{-3}$$

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Problem 2 Solution

2. Find the derivatives of the following functions using the basic rules. Leave your answers in an unsimplified form so that it is clear what method you used.

(a) $x^5 + x^{-1/4} + 19$

(b) $(x^4 + x)e^x$

(c) $\dfrac{2x+1}{3x+2}$

Solution:

(a) Use the Power Rule.

$$(x^5 + x^{-1/4} + 19)' = \boxed{5x^4 - \frac{1}{4}x^{-5/4}}$$

(b) Use the Product Rule.

$$\begin{aligned}[(x^4 + x)e^x]' &= (x^4 + x)(e^x)' + (x^4 + x)'e^x \\ &= \boxed{(x^4 + x)e^x + (4x^3 + 1)e^x}\end{aligned}$$

(c) Use the Quotient Rule.

$$\begin{aligned}\left(\frac{2x+1}{3x+2}\right)' &= \frac{(3x+2)(2x+1)' - (2x+1)(3x+2)'}{(3x+2)^2} \\ &= \boxed{\frac{2(3x+2) - 3(2x+1)}{(3x+2)^2}}\end{aligned}$$

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Problem 3 Solution

3. Find the equation of the tangent line to $y = 1 + \sqrt{x}$ at $x = 25$.

Solution: The derivative y' is found using the Power Rule.

$$y' = \left(1 + \sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$$

At $x = 25$ the values of y and y' are:

$$y(25) = 1 + \sqrt{25} = 6$$
$$y'(25) = \frac{1}{2\sqrt{25}} = \frac{1}{10}$$

We now know that the point $(25, 6)$ is on the tangent line and that the slope of the tangent line is $\frac{1}{10}$. Therefore, an equation for the tangent line in point-slope form is:

$$\boxed{y - 6 = \frac{1}{10}(x - 25)}$$

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Problem 4 Solution

4. Find a specific value for δ such that, if $|x-3| < \delta$, then $|2x-6| < 0.01$.

Solution: Working with the inequality $|2x-6| < 0.01$ we have:

$$\begin{aligned} |2x-6| &< 0.01 \\ 2|x-3| &< 0.01 \\ |x-3| &< 0.005 \end{aligned}$$

Thus, we choose $\delta = 0.005$. This guarantees that if $|x-3| < 0.005$ then $|2x-6| < 0.01$.

Note that we can choose any δ that is less than or equal to 0.005.

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Problem 5 Solution

5. Suppose that

$$f(4) = 4, \quad f'(4) = -2$$
$$g(4) = 5, \quad g'(4) = -3$$

Find the derivative of the quotient function $\dfrac{f(x)}{g(x)}$ at $x = 4$.

Solution: Using the Quotient Rule we have:

$$\left[\frac{f(x)}{g(x)}\right]' = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

At $x = 4$, the value of the derivative is:

$$\begin{aligned}\left[\frac{f(x)}{g(x)}\right]'\Bigg|_{x=4} &= \frac{g(4)f'(4) - f(4)g'(4)}{[g(4)]^2} \\ &= \frac{(5)(-2) - (4)(-3)}{5^2} \\ &= \boxed{\frac{2}{25}}\end{aligned}$$

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Problem 6 Solution

6. The table below shows some values of functions $f(x)$, $g(x)$, and $h(x)$ in the interval $1 \leq x \leq 3$.

x	1	1.5	2	2.5	3
$f(x)$	0.25	0.75	0.75	0.25	-0.75
$g(x)$	0.75	0.25	-0.25	-0.75	-1.25
$h(x)$	1.5	0.5	-0.5	-1.5	-2.5

(a) Calculate the average rate of change of $f(x)$ on the interval $1 \leq x \leq 3$.

(b) Which one of the functions g and h is the derivative of f? Explain your answer by citing some feature of the data.

Solution:

(a) The average rate of change formula is:

$$\text{average ROC} = \frac{f(b) - f(a)}{b - a}$$

where $b = 3$ and $a = 1$. Therefore, the average rate of change of $f(x)$ on the interval is:

$$\text{average ROC} = \frac{f(3) - f(1)}{3 - 1} = \frac{-0.75 - 0.25}{2} = \boxed{-0.5}$$

(b) To estimate the derivative $f'(2)$ we use the formula:

$$f'(2) \approx \frac{f(x) - f(2)}{x - 2}$$

Choosing $x = 2.5$ we get the estimate:

$$f'(2) \approx \frac{f(2.5) - f(2)}{2.5 - 2} = \frac{0.25 - 0.75}{0.5} = -1$$

Choosing $x = 1.5$ we get the estimate:

$$f'(2) \approx \frac{f(1.5) - f(2)}{1.5 - 2} = \frac{0.75 - 0.75}{-0.5} = 0$$

The average of these two estimates is:

$$\text{average estimate of } f'(2) = \frac{-1 + 0}{2} = -0.5$$

Note that this is the value of $h(2)$. Therefore, it appears that h is the derivative of f.