Math 180, Final Exam, Study Guide
Problem 1 Solution

1. Differentiate with respect to x. Write your answers showing the use of the appropriate techniques. Do *not* simplify.

(a) $x^{1066} + x^{1/2} - x^{-2}$ (b) $e^{\sqrt{x}}$ (c) $\dfrac{\sin(x)}{5+x^2}$

Solution:

(a) Use the Power Rule.

$$(x^{1066} + x^{1/2} - x^{-2})' = \boxed{1066x^{1065} + \frac{1}{2}x^{-1/2} + 2x^{-3}}$$

(b) Use the Chain Rule.

$$\begin{aligned}(e^{\sqrt{x}})' &= e^{\sqrt{x}} \cdot \left(\sqrt{x}\right)' \\ &= \boxed{e^{\sqrt{x}} \cdot \left(\frac{1}{2\sqrt{x}}\right)}\end{aligned}$$

(c) Use the Quotient Rule.

$$\begin{aligned}\left(\frac{\sin(x)}{5+x^2}\right)' &= \frac{(5+x^2)(\sin(x))' - \sin(x)(5+x^2)'}{(5+x^2)^2} \\ &= \boxed{\frac{(5+x^2)\cos(x) - \sin(x)(2x)}{(5+x^2)^2}}\end{aligned}$$

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Problem 2 Solution

2. Differentiate with respect to x. Write your answers showing the use of the appropriate techniques. Do *not* simplify.

(a) $e^{3x}\cos(5x)$ (b) $\ln(x^2+x+1)$ (c) $\tan\left(\frac{1}{x}\right)$

Solution:

(a) Use the Product and Chain Rules.

$$\begin{aligned}\left[e^{3x}\cos(5x)\right]' &= e^{3x}\left[\cos(5x)\right]' + \left(e^{3x}\right)'\cos(5x)\\ &= \boxed{e^{3x}\left[-5\sin(5x)\right] + 3e^{3x}\cos(5x)}\end{aligned}$$

(b) Use the Chain Rule.

$$\begin{aligned}\left[\ln(x^2+x+1)\right]' &= \frac{1}{x^2+x+1}\cdot(x^2+x+1)'\\ &= \boxed{\frac{1}{x^2+x+1}\cdot(2x+1)}\end{aligned}$$

(c) Use the Chain Rule.

$$\begin{aligned}\left[\tan\left(\frac{1}{x}\right)\right]' &= \sec^2\left(\frac{1}{x}\right)\cdot\left(\frac{1}{x}\right)'\\ &= \boxed{\sec^2\left(\frac{1}{x}\right)\cdot\left(-\frac{1}{x^2}\right)}\end{aligned}$$

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Problem 3 Solution

3. Differentiate with respect to x. Write your answers showing the use of the appropriate techniques. Do *not* simplify.

(a) $x^{2005} + x^{2/3}$ (b) $\cos(\pi x)$ (c) $\dfrac{1+2x}{3+x^2}$

Solution:

(a) Use the Power Rule.

$$(x^{2005} + x^{2/3})' = \boxed{2005x^{2004} + \frac{2}{3}x^{-1/3}}$$

(b) Use the Chain Rule.

$$\begin{aligned}[\cos(\pi x)]' &= -\sin(\pi x) \cdot (\pi x)' \\ &= \boxed{-\sin(\pi x) \cdot (\pi)}\end{aligned}$$

(c) Use the Quotient Rule.

$$\begin{aligned}\left(\frac{1+2x}{3+x^2}\right)' &= \frac{(3+x^2)(1+2x)' - (1+2x)(3+x^2)'}{(3+x^2)^2} \\ &= \boxed{\frac{(3+x^2)(2) - (1+2x)(2x)}{(3+x^2)^2}}\end{aligned}$$

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Problem 4 Solution

4. Differentiate with respect to x. Write your answers showing the use of the appropriate techniques. Do *not* simplify.

(a) x^2e^{-3x} (b) $\arctan(x)$ (c) $\ln(\cos(x))$

Solution:

(a) Use the Product and Chain Rules.

$$\begin{aligned}(x^2e^{-3x})' &= x^2(e^{-3x})' + (x^2)'e^{-3x} \\ &= \boxed{-3x^2e^{-3x} + 2xe^{-3x}}\end{aligned}$$

(b) This is a basic derivative.

$$(\arctan(x))' = \boxed{\frac{1}{1+x^2}}$$

(c) Use the Chain Rule.

$$\begin{aligned}[\ln(\cos(x))]' &= \frac{1}{\cos(x)} \cdot (\cos(x))' \\ &= \boxed{\frac{1}{\cos(x)} \cdot (-\sin(x))}\end{aligned}$$

Math 180, Final Exam, Study Guide
Problem 5 Solution

5. Use implicit differentiation to find the slope of the line tangent to the curve

$$x^2 + xy + y^2 = 7$$

at the point $(2, 1)$.

Solution: We must find $\dfrac{dy}{dx}$ using implicit differentiation.

$$\begin{aligned}
x^2 + xy + y^2 &= 7 \\
\frac{d}{dx}x^2 + \frac{d}{dx}(xy) + \frac{d}{dx}y^2 &= \frac{d}{dx}7 \\
2x + \left(x\frac{dy}{dx} + y\right) + 2y\frac{dy}{dx} &= 0 \\
x\frac{dy}{dx} + 2y\frac{dy}{dx} &= -2x - y \\
\frac{dy}{dx}(x + 2y) &= -2x - y \\
\frac{dy}{dx} &= \frac{-2x - y}{x + 2y}
\end{aligned}$$

The value of $\dfrac{dy}{dx}$ at $(2, 1)$ is the slope of the tangent line.

$$\left.\frac{dy}{dx}\right|_{(2,1)} = \frac{-2(2) - 1}{2 + 2(1)} = -\frac{5}{4}$$

An equation for the tangent line at $(2, 1)$ is then:

$$\boxed{y - 1 = -\frac{5}{4}(x - 2)}$$

Math 180, Final Exam, Study Guide
Problem 6 Solution

6. Use calculus to find the exact x- and y-coordinates of any local maxima, local minima, and inflection points of the function $f(x) = x^3 - 12x + 5$.

Solution: The critical points of $f(x)$ are the values of x for which either $f'(x)$ does not exist or $f'(x) = 0$. Since $f(x)$ is a polynomial, $f'(x)$ exists for all $x \in \mathbb{R}$ so the only critical points are solutions to $f'(x) = 0$.

$$\begin{aligned} f'(x) &= 0 \\ \left(x^3 - 12x + 5\right)' &= 0 \\ 3x^2 - 12 &= 0 \\ 3(x^2 - 4) &= 0 \\ 3(x-2)(x+2) &= 0 \\ x &= \pm 2 \end{aligned}$$

Thus, $\boxed{x = \pm 2}$ are the critical points of f. We will use the First Derivative Test to classify the points as either local maxima or a local minima. We take the domain of $f(x)$ and split it into the intervals $(-\infty, -2)$, $(-2, 2)$, and $(2, \infty)$ and then evaluate $f'(x)$ at a test point in each interval.

Interval	**Test Number,** c	$f'(c)$	**Sign of** $f'(c)$
$(-\infty, -2)$	-3	$f'(-3) = 15$	$+$
$(-2, 2)$	0	$f'(0) = -12$	$-$
$(2, \infty)$	3	$f'(3) = 15$	$+$

Since the sign of $f'(x)$ changes sign from $+$ to $-$ at $x = -2$, the point $f(-2) = 21$ is a local maximum and since the sign of $f'(x)$ changes from $-$ to $+$ at $x = 2$, the point $f(2) = -11$ is a local minimum.

The critical points of $f(x)$ are the values of x where $f''(x)$ changes sign. To determine these we first find the values of x for which $f''(x) = 0$.

$$\begin{aligned} f''(x) &= 0 \\ \left(3x^2 - 12\right)' &= 0 \\ 6x &= 0 \\ x &= 0 \end{aligned}$$

We now take the domain of $f(x)$ and split it into the intervals $(-\infty, 0)$ and $(0, \infty)$ and then evaluate $f''(x)$ at a test point in each interval.

Interval	**Test Number,** c	$f''(c)$	**Sign of** $f''(c)$
$(-\infty, 0)$	-1	$f''(-1) = -6$	$-$
$(0, \infty)$	1	$f''(1) = 6$	$+$

We see that $f''(x)$ changes sign at $x = 0$. Thus, $x = 0$ is an inflection point.

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Problem 7 Solution

7. Use calculus to find the x- and y-coordinates of any local maxima, local minima, and inflection points of the function $f(x) = xe^{-x}$ on the interval $0 \le x < \infty$. The y-coordinates may be written in terms of e or as a 4-place decimal.

Solution: The critical points of $f(x)$ are the values of x for which either $f'(x)$ does not exist or $f'(x) = 0$. Since $f(x)$ is a polynomial, $f'(x)$ exists for all $x \in \mathbb{R}$ so the only critical points are solutions to $f'(x) = 0$.

$$\begin{aligned} f'(x) &= 0 \\ \left(xe^{-x}\right)' &= 0 \\ -xe^{-x} + e^{-x} &= 0 \\ e^{-x}(-x+1) &= 0 \\ -x+1 &= 0 \\ x &= 1 \end{aligned}$$

Thus, $\boxed{x = 1}$ is the only critical point of f. We will use the Second Derivative Test to classify it.

$$f''(x) = -e^{-x} + xe^{-x} - e^{-x} = e^{-x}(x-2)$$

At $x = 1$ we have $f''(1) = -e^{-1} < 0$. Thus, the Second Derivative Test implies that $f(1) = e^{-1}$ is a local maximum.

The critical points of $f(x)$ are the values of x where $f''(x)$ changes sign. To determine these we first find the values of x for which $f''(x) = 0$.

$$\begin{aligned} f''(x) &= 0 \\ e^{-x}(x-2) &= 0 \\ x-2 &= 0 \\ x &= 2 \end{aligned}$$

We now take the domain of $f(x)$ and split it into the intervals $(-\infty, 2)$ and $(2, \infty)$ and then evaluate $f''(x)$ at a test point in each interval.

Interval	**Test Number, c**	$f''(c)$	**Sign of $f''(c)$**
$(-\infty, 2)$	0	$f''(0) = -2$	$-$
$(2, \infty)$	3	$f''(3) = e^{-3}$	$+$

We see that $f''(x)$ changes sign at $x = 2$. Thus, $x = 2$ is an inflection point. The corresponding value of f is $f(2) = 2e^{-2}$.

Math 180, Final Exam, Study guide
Problem 8 Solution

8. Estimate the integral $\int_0^{40} f(t)\,dt$ using the left Riemann sum with four subdivisions. Some values of the function f are given in the table:

t	0	10	20	30	40
$f(t)$	5.3	5.1	4.6	3.7	2.3

If the function f is known to be decreasing, could the integral be larger than your estimate? Explain why or why not.

Solution: In calculating L_4, the value of Δx is:

$$\Delta x = \frac{b-a}{N} = \frac{40-0}{4} = 10$$

The integral estimates are then:

$$\begin{aligned} L_4 &= \Delta x\,[f(0) + f(10) + f(20) + f(30)] \\ &= 10\,[5.3 + 5.1 + 4.6 + 3.7] \\ &= \boxed{187} \end{aligned}$$

Since f is known to be decreasing, we know that $R_4 \leq S \leq L_4$ where S is the actual value of the integral. Therefore, the actual value of the integral cannot be larger than L_4.

Math 180, Final Exam, Study Guide
Problem 9 Solution

9. Write the integral which gives the area of the region between $x = 0$ and $x = 2$, above the x-axis, and below the curve $y = 9 - x^2$. Evaluate your integral exactly to find the area.

Solution: The area of the region is given by the integral:

$$\int_0^2 (9 - x^2)\, dx$$

We use FTC I to evaluate the integral.

$$\begin{aligned}\int_0^2 (9 - x^2)\, dx &= 9x - \frac{x^3}{3}\bigg|_0^2 \\ &= \left(9(2) - \frac{2^3}{3}\right) - \left(9(0) - \frac{0^3}{3}\right) \\ &= \boxed{\frac{46}{3}}\end{aligned}$$

Math 180, Final Exam, Study Guide
Problem 10 Solution

10. Write the integral which gives the area of the region between $x = 1$ and $x = 3$, above the x-axis, and below the curve $y = x - \dfrac{1}{x^2}$. Evaluate your integral exactly to find the area.

Solution: The area is given by the integral:

$$\int_1^3 \left(x - \frac{1}{x^2}\right)\, dx$$

Using FTC I, we have:

$$\begin{aligned}
\int_1^3 \left(x - \frac{1}{x^2}\right)\, dx &= \frac{x^2}{2} + \frac{1}{x}\bigg|_1^3 \\
&= \left(\frac{3^2}{2} + \frac{1}{3}\right) - \left(\frac{1^2}{2} + \frac{1}{1}\right) \\
&= \boxed{\frac{10}{3}}
\end{aligned}$$

Math 180, Final Exam, Study Guide
Problem 11 Solution

11. The average value of the function $f(x)$ on the interval $a \leq x \leq b$ is

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

Find the average value of the function $f(x) = \dfrac{1}{x^2}$ on the interval $2 \leq x \leq 6$.

Solution: The average value is

$$\begin{aligned}\frac{1}{6-2}\int_2^6 \frac{1}{x^2}\,dx &= \frac{1}{4}\left[-\frac{1}{x}\right]_2^6 \\ &= \frac{1}{4}\left[-\frac{1}{6} - \left(-\frac{1}{2}\right)\right] \\ &= \boxed{\frac{1}{12}}\end{aligned}$$

Math 180, Final Exam, Study Guide
Problem 12 Solution

12. Find

$$\lim_{x\to 0} \frac{\sqrt{1+x}-1}{x}$$

Explain how you obtain your answer.

Solution: Upon substituting $x = 0$ into the function $f(x) = \dfrac{\sqrt{1+x}-1}{x}$ we find that

$$\frac{\sqrt{1+x}-1}{x} = \frac{\sqrt{1+0}-1}{0} = \frac{0}{0}$$

which is indeterminate. We can resolve the indeterminacy by multiplying $f(x)$ by the "conjugate" of the numerator divided by itself.

$$\begin{aligned}
\lim_{x\to 0} \frac{\sqrt{1+x}-1}{x} &= \lim_{x\to 0} \frac{\sqrt{1+x}-1}{x} \cdot \frac{\sqrt{1+x}+1}{\sqrt{1+x}+1} \\
&= \lim_{x\to 0} \frac{(1+x)-1}{x(\sqrt{1+x}+1)} \\
&= \lim_{x\to 0} \frac{x}{x(\sqrt{1+x}+1)} \\
&= \lim_{x\to 0} \frac{1}{\sqrt{1+x}+1} \\
&= \frac{1}{\sqrt{1+0}+1} \\
&= \boxed{\frac{1}{2}}
\end{aligned}$$

Math 180, Final Exam, Study Guide
Problem 13 Solution

13. Find

$$\lim_{x\to 0} \frac{1-\cos(3x)}{x^2}$$

Explain how you obtain your answer.

Solution: Upon substituting $x = 0$ into the function we find that

$$\frac{1-\cos(3x)}{x^2} = \frac{1-\cos(3\cdot 0)}{0^2} = \frac{0}{0}$$

which is indeterminate. We resolve this indeterminacy by using L'Hôpital's Rule.

$$\begin{aligned}\lim_{x\to 0} \frac{1-\cos(3x)}{x^2} &\overset{\text{L'H}}{=} \lim_{x\to 0} \frac{(1-\cos(3x))'}{(x^2)'} \\ &= \lim_{x\to 0} \frac{3\sin(3x)}{2x} \\ &= \frac{3}{2} \lim_{x\to 0} \frac{\sin(3x)}{x}\end{aligned}$$

This limit has the indeterminate form $\frac{0}{0}$ so we use L'Hôpital's rule again.

$$\begin{aligned}\frac{3}{2} \lim_{x\to 0} \frac{\sin(3x)}{x} &\overset{\text{L'H}}{=} \frac{3}{2} \lim_{x\to 0} \frac{(\sin(3x))'}{(x)'} \\ &= \frac{3}{2} \lim_{x\to 0} \frac{3\cos(3x)}{1} \\ &= \frac{3}{2} \cdot \frac{3\cos(3\cdot 0)}{1} \\ &= \boxed{\frac{9}{2}}\end{aligned}$$

Math 180, Exam 2, Study Guide
Problem 14 Solution

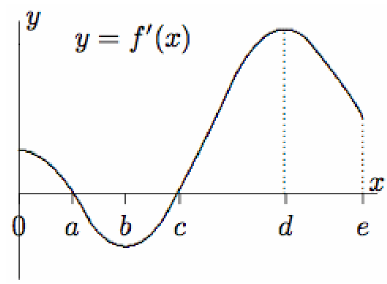
14. The graph below represents the derivative, $f'(x)$.

(i) On what interval is the original f decreasing?

(ii) At which labeled value of x is the value of $f(x)$ a global minimum?

(iii) At which labeled value of x is the value of $f(x)$ a global maximum?

(iv) At which labeled values of x does $y = f(x)$ have an inflection point?



Solution:

(i) f is decreasing when $f'(x) < 0$. From the graph, we can see that $f'(x) < 0$ on the interval (a, c).

(ii) We know that $f(x) = f(0) + \int_0^x f'(t)\,dt$. That is, $f(x)$ is the signed area between $y = f'(x)$ and the x-axis on the interval $[0, x]$ plus a constant. Thus, the global minimum of $f(x)$ will occur when the signed area is a minimum. This occurs at $x = c$.

(iii) The global maximum of $f(x)$ will occur when the signed area is a maximum. This occurs at $x = e$.

(iv) An inflection point occurs when $f''(x)$ changes sign, i.e. when $f'(x)$ transitions from increasing to decreasing or vice versa. This occurs at $x = b$ and $x = d$.

Math 180, Final Exam, Study Guide
Problem 15 Solution

15. The function $f(x)$ has the following properties:

- $f(5) = 2$
- $f'(5) = 0.6$
- $f''(5) = -0.4$

(a) Find the tangent line to $y = f(x)$ at the point $(5, 2)$.

(b) Use (a) to estimate $f(5.2)$.

(c) If f is known to be concave down, could your estimate in (b) be greater than the actual $f(5.2)$? Give a reason supporting your answer.

Solution:

(a) The slope of the tangent line at the point $(5, 2)$ is $f'(5) = 0.6$. Thus, an equation for the tangent line is:

$$\boxed{y - 2 = 0.6(x - 5)}$$

(b) The tangent line gives the linearization of $f(x)$ at $x = 2$. That is,

$$L(x) = 2 + 0.6(x - 5)$$

Thus, an approximate value of $f(5.2)$ using the linearization is:

$$f(5.2) \approx L(5.2) = 2 + 0.6(5.2 - 5) = \boxed{2.12}$$

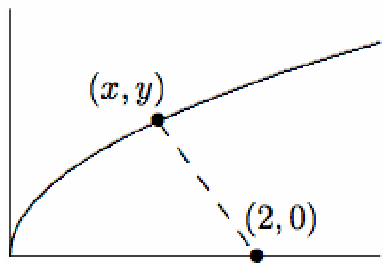
(c) If f is concave down then the tangent line at $x = 5$ is always above the graph of $y = f(x)$ except at $x = 5$. Thus, if we use the tangent line to approximate $f(5.2)$, the estimate will give us a value that is greater than the actual value of $f(5.2)$.

Math 180, Final Exam, Study Guide
Problem 16 Solution

16. The point (x, y) lies on the curve $y = \sqrt{x}$.

(a) Find the distance from (x, y) to $(2, 0)$ as a function $f(x)$ of x alone.

(b) Find the value of x that makes this distance the smallest.



Solution: The function we seek to minimize is the distance between (x, y) and $(2, 0)$.

Function : $\quad$ Distance $= \sqrt{(x-2)^2 + (y-0)^2}$ $\quad$ (1)

The constraint in this problem is that the point (x, y) must lie on the curve $y = \sqrt{x}$.

Constraint : $\quad y = \sqrt{x}$ $\quad$ (2)

Plugging this into the distance function (1) and simplifying we get:

$$\text{Distance} = \sqrt{(x-2)^2 + \left(\sqrt{x} - 0\right)^2}$$
$$f(x) = \sqrt{x^2 - 3x + 4}$$

We want to find the absolute minimum of $f(x)$ on the **interval** $[0, \infty)$. We choose this interval because (x, y) must be on the line $y = \sqrt{x}$ and the domain of this function is $[0, \infty)$.

The absolute minimum of $f(x)$ will occur either at a critical point of $f(x)$ in $(0, \infty)$, at $x = 0$, or it will not exist. The critical points of $f(x)$ are solutions to $f'(x) = 0$.

$$f'(x) = 0$$
$$\left[\left(x^2 - 3x + 4\right)^{1/2}\right]' = 0$$
$$\frac{1}{2}\left(x^2 - 3x + 4\right)^{-1/2} \cdot \left(x^2 - 3x + 4\right)' = 0$$
$$\frac{2x - 3}{2\sqrt{x^2 - 3x + 4}} = 0$$
$$2x - 3 = 0$$
$$x = \frac{3}{2}$$

Plugging this into $f(x)$ we get:

$$f\left(\frac{3}{2}\right) = \sqrt{\left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) + 4} = \frac{\sqrt{7}}{2}$$

Evaluating $f(x)$ at $x = 0$ and taking the limit as $x \to \infty$ we get:

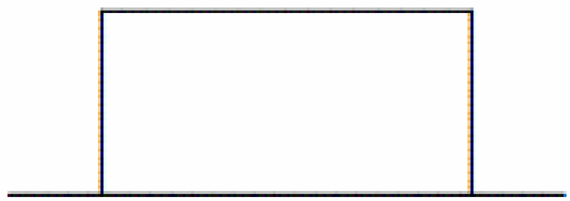
$$f(0) = \sqrt{0^2 - 3(0) + 4} = 2$$
$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \sqrt{x^2 - 3x + 4} = \infty$$

both of which are larger than $\frac{\sqrt{7}}{2}$. We conclude that the distance is an absolute minimum at $\boxed{x = \frac{3}{2}}$ and that the resulting distance is $\frac{\sqrt{7}}{2}$. The last step is to find the corresponding value for y by plugging $x = \frac{3}{2}$ into equation (2).

$$\boxed{y = \sqrt{\frac{3}{2}}}$$

Math 180, Final Exam, Study Guide
Problem 17 Solution

17. You have 24 feet of rabbit-proof fence to build a rectangular garden using one wall of a house as one side of the garden and the fence on the other three sides. What dimensions of the rectangle give the largest possible area for the garden?



Solution: We begin by letting x be the length of the side opposite the house and y be the lengths of the remaining two sides. The function we seek to minimize is the area of the garden:

$$\textbf{Function}: \quad \text{Area} = xy \tag{1}$$

The constraint in this problem is that the length of the fence is 24 feet.

$$\textbf{Constraint}: \quad x + 2y = 24 \tag{2}$$

Solving the constraint equation (2) for y we get:

$$y = 12 - \frac{x}{2} \tag{3}$$

Plugging this into the function (1) and simplifying we get:

$$\begin{aligned} \text{Area} &= x\left(12 - \frac{x}{2}\right) \\ f(x) &= 12x - \frac{1}{2}x^2 \end{aligned}$$

We want to find the absolute maximum of $f(x)$ on the **interval** $[0, 24]$.

The absolute maximum of $f(x)$ will occur either at a critical point of $f(x)$ in $[0, 24]$ or at one of the endpoints of the interval. The critical points of $f(x)$ are solutions to $f'(x) = 0$.

$$\begin{aligned} f'(x) &= 0 \\ \left(12x - \frac{1}{2}x^2\right)' &= 0 \\ 12 - x &= 0 \\ x &= 12 \end{aligned}$$

Plugging this into $f(x)$ we get:

$$f(12) = 12(12) - \frac{1}{2}(12)^2 = 72$$

Evaluating $f(x)$ at the endpoints we get:

$$f(0) = 12(0) - \frac{1}{2}(0)^2 = 0$$
$$f(24) = 12(24) - \frac{1}{2}(24)^2 = 0$$

both of which are smaller than 72. We conclude that the area is an absolute maximum at $\boxed{x = 12}$ and that the resulting area is 72. The last step is to find the corresponding value for y by plugging $x = 12$ into equation (3).

$$y = 12 - \frac{x}{2} = 12 - \frac{12}{2} = \boxed{6}$$

Math 180, Final Exam, Study Guide
Problem 18 Solution

18. Evaluate the integral $\int xe^{x^2-1}\,dx$.

Solution: We use the substitution $u = x^2 - 1$, $\frac{1}{2}\,du = x\,dx$. Making the substitutions and evaluating the integral we get:

$$\begin{aligned} \int xe^{x^2-1}\,dx &= \frac{1}{2}\int e^u\,du \\ &= \frac{1}{2}e^u + C \\ &= \boxed{\frac{1}{2}e^{x^2-1} + C} \end{aligned}$$

Math 180, Final Exam, Study Guide
Problem 19 Solution

19. Evaluate the integral $\int \sin^2 \cos x \, dx$.

Solution: We use the substitution $u = \sin x$, $du = \cos x \, dx$. Making the substitutions and evaluating the integral we get:

$$\begin{aligned}
\int \sin^2 x \cos x \, dx &= \int u^2 \, du \\
&= \frac{u^3}{3} + C \\
&= \boxed{\frac{\sin^3 x}{3} + C}
\end{aligned}$$

Math 180, Final Exam, Study Guide
Problem 20 Solution

20. Evaluate $\displaystyle\int_2^5 \frac{2x-3}{\sqrt{x^2-3x+6}}\,dx$.

Solution: We use the substitution $u = x^2-3x+6$, $du = (2x-3)\,dx$. The limits of integration become $u = 2^2-3(2)+6 = 4$ and $u = 5^2-3(5)+6 = 16$. Making the substitutions and evaluating the integral we get:

$$\begin{aligned}
\int_2^5 \frac{2x-3}{\sqrt{x^2-3x+6}}\,dx &= \int_4^{16} \frac{1}{\sqrt{u}}\,du \\
&= 2\sqrt{u}\,\Big|_4^{16} \\
&= 2\sqrt{16} - 2\sqrt{4} \\
&= \boxed{4}
\end{aligned}$$