MCS521-Spring 2017
HOMEWORK ASSIGNMENTS

MCS 521, Fall 2017
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1 HOMEWORK ASSIGNMENT 1 Assigned 9-5 – Due 9-19-2015

Do three problems from the following problems. (If you plan to take the combinatorics prelim, try to do all theoretical problems.)

1. From [1] p.p. 334-335, *Exercises*: A1, A2, A6, A9.
2. Prove Corollary A.6 from Theorem A.5 by doing the following substitutions in the statement Corollary A.6: Call $y = -x', x = y', A' = A^\top, b' = -c, c' = b$.
3. *Programming problem*: Write a code, (Matlab or other software is fine), to do the following:

   Input integers $n, m \geq 1$, matrix $A = [a_{ij}]_{i=j=1}^{m,n} \in \mathbb{R}^n$ column vector $\mathbf{b} = (b_1, \ldots, b_m)^\top \in \mathbb{R}^m$.

   Set $n(1) = n, m(1) = m, \mathbf{b}(1) = \mathbf{b}, \mathbf{x}(1) = \mathbf{x}, A(1) = A, k = 1$. Consider the system

$$A(k)\mathbf{x}(k) \leq \mathbf{b}(k), \quad A(k) = [a(k)_{ij}]_{i=j=1}^{m(k),n(k)} \in \mathbb{R}^{m(k)\times n(k)}, \tag{1.1}$$
$$\mathbf{x}(k) = (x_{1,k}, \ldots, x_{n(k),k})^\top \in \mathbb{R}^{n(k)}, \mathbf{b}(k) = (b_{1,k}, \ldots, b_{m(k),k})^\top \in \mathbb{R}^{m(k)}.$$

   (a) Scan the columns of $A(k)$ and pick up a column which has $p = m' \geq 1$ and $q = m'' - m' \geq 1$ positive and negative elements respectively, such that $(p-1)(q-1)$ is minimal.

   If such column does not exist, then decide if the system solvable or not. (It would be unsolvable if and only if there exists a zero row i of $A(k)$ and $b_{i,k} < 0$.) Print solvable, if the system solvable, or unsolvable if the system unsolvable. Print $n, k, n-k$ and $m(k)$ and exit the program.

   If $n(k) = 1$ decide if the system (1.1) in one variable is solvable. Print solvable, if the system solvable, or unsolvable if the system unsolvable. Print $n, k, n-k$ and $m(k)$ and exit the program.

(b) Interchange the rows $A(k)$ and $\mathbf{b}(k)$ and the columns of $A(k)$, and divide the m'' rows of $A(k)$ by positive numbers so that you have the situation of Fourier-Motzkin elimination elimination as on page 326 of [1].

(c) Perform the Fourier-Motzkin elimination as on page 326 of [1]. Set $s = k + 1$ and call the new system with $n(k) - 1$ variables $A(s)\mathbf{x}(s) \leq \mathbf{b}(s)$. Set $k = s$. Go to (a).

Check your program on a number of simple examples with one or two variables, $(n = 1, 2)$, and $m = 2, 3, 4$, that it works correctly.

Now perform the following simulations.

Choose the entries of A and $\mathbf{b}$ at random for $n = 1, \ldots, 10$ and $m = \lceil \frac{n}{2} \rceil$ to $2n$. (You can choose the entries of A and $\mathbf{b}$ random integers in $[-100, 100]$.)

First verify that for $m \leq n + 1$ usually the inequalities $A\mathbf{x} \leq \mathbf{b}$ are solvable. (Why?)

Next run these simulations a number of time to get the statistics of the complexity of Fourier-Motzkin elimination. For a given values of n and m what was the maximum $m(k)$, (the one the program printed out when it ended). What was the average of the values of $m(k)$, which where printed out, you got. This will give an idea of the maximum complexity and the average complexity of the Fourier-Motzkin elimination method. Print the table: n horizontally, m vertical and in the place (m, n) print the maximum and the average value of $m(k)$.

2 HOMEWORK ASSIGNMENT 2 Assigned 9-19 – Due 10-3-2015

Do three problems from [1, §2.1]: 2.1, 2.6, 2.7, 2.8, 2.9, 2.10, 2.16.
Do three problems from [1, §2.2]: 2.18, 2.21, 2.22, 2.23, 2.34, 2.35. (If you plan to take the combinatorics prelim, try to do 5 problems from each section.)

3 HOMEWORK ASSIGNMENT 3 Assigned 10-5 Due 10-24-2017

Do three problems from [1, §3.2]: Exercises: 3.2, 3.3, 3.4, 3.5, 3.6, 3.7, 3.8, 3.9.
Do four problems from [1, §3.3]: Exercises: 3.17, 3.18, 3.21, 3.23, 3.24, 3.26, 3.31, 3.35.

4 HOMEWORK ASSIGNMENT 4 Assigned 10-24 Due 11-9-2017

Do three problems from [1, §4.1]: *Exercises*: 4.3, 4.4, 4.5, 4.6, 4.10, 4.11.
Do four problems from [1, §3.5]: *Exercises*: 3.52, 3.53, 3.54, 3.56, 3.60, 3.66, 3.67.

5 HOMEWORK ASSIGNMENT 5
Assigned 11-9 Due 11-21-2017

Do three problems from Aliabadi's lecture 10-31 and 11-2:

1. Alternative proof of Hall's Theorem: Consider a bipartite graph G with bipartition X, Y, satisfying $|N(S)| \geq |S|$, for any $S \subseteq X$. Use induction on $|X|$ to prove that G has a matching that saturates X.

2. Prove or disprove: Every tree has at most one perfect matching.

3. Determine the minimum size of a maximal matching in the cycle C_n.

4. If we denote by $\Omega_{n,s}$ the subset of symmetric doubly stochastic matrices, show that each $A \in \Omega_{n,s}$ can be written as a convex combination of $\frac{1}{2}(P + P^T)$, where $P \in \mathcal{P}_n$.

5. Prove that a bipartite graph G has a perfect matching iff $|N(S)| \geq |S|$, for any $S \subseteq V(G)$, and present an infinite class of examples to prove that this characterization does not hold for all graphs.

Do four problems from [1], *Exercises*: 5.1, 5.2, 5.4, 5.6, 5.7, 5.9, 5.13, 5.14, 5.18.

6 HOMEWORK ASSIGNMENT 6
Assigned 11-22 Due 12-7-2017

Part I -Problems from [1] - Do 3 problems out of the following ones:
Problem 1: Suppose that the Slither game ([1, Problem 5.18]) is played on a simple connected graph G, such that each vertex of G is inessential. Show that the Second player has the winning strategy.
Exercises: 5.22, 5.40, 5.43, 5.44, 5.45.

Part II - Problems on SDP will be assigned later.

References

[1] W.J. Cook, W.H. Cunningham, W.R. Pulleyblank, A.Schrijver, *Combinatorial Optimization*, Wiley, 1998.