Transverse noncommutative geometry of foliations

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We define an L^2-signature for proper actions on spaces of leaves of transversely oriented foliations with bounded geometry. This is achieved by using the Connes fibration to reduce the problem to the case of Riemannian bifoliations where we show that any transversely elliptic first order operator in an appropriate Beals-Greiner calculus, satisfying the usual axioms, gives rise to a semi-finite spectral triple over the crossed product algebra of the foliation by the action, and hence a periodic cyclic cohomology class through the Connes-Chern character. The Connes-Moscovici hypoelliptic signature operator yields an example of such a triple and gives the differential definition of our L^2-signature. For Galois coverings of bounded geometry foliations, we also define an Atiyah-Connes semi-finite spectral triple which generalizes to Riemannian bifoliations the Atiyah approach to the L^2-index theorem. The compatibility of the two spectral triples with respect to Morita equivalence is proven, and by using an Atiyah-type theorem proven in "Atiyah covering index theorem for Riemannian foliations", we deduce some integrality results for Riemannian foliations with torsion-free monodromy groupoids.