Math 313 Homework 1
Due Friday January 25

Q1 Prove carefully, making clear at each step which axioms you are using, that in an ordered field, $-1 < 0$. (*I.e.*, $-1 \leq 0$ & $-1 \neq 0$.) Then use Theorem 3.2 (iv) to deduce that the complex numbers $\mathbb{C}$ cannot be an ordered field.

For discussion Do you think that field $\mathbb{Q}(t)$ of rational functions (i.e. fractions with polynomial numerator and denominator) having rational number coefficients can be an ordered field?

Q2 Suppose x is a rational number such that $x^3 + ax^2 + bx + 1 = 0$ for some integers a and b. Show that x must be $+1$ or -1, and furthermore that either $a = b$ or $a + b = -2$.

Q3 Define a sequence of natural numbers $a_1, a_2, \ldots, a_n, \ldots$ as follows: $a_1 = 1$, and once given a_n, define a_{n+1} to be $2a_n + 1$. (Such a definition is known as a recursive definition.) Prove that all $n \in \mathbb{N}$, we have $a_n = 2^n - 1$.

Q4 Ross, exercise 3.4.

Q5 Ross, exercise 4.6