Math 313 Homework 2
Due Friday January 31

Q1 Ross 8.2 a), b), e)

Q2 Ross 8.9

Q3 Suppose $d_1, d_2, d_3, \ldots$ is a sequence such that $\lim_{n\to\infty} d_n = L$ where L is finite. Let $e_n = \frac{d_n + d_{n+1}}{2}$ for all n. Show that $\lim_{n\to\infty} e_n = L$ as well.

Extra Credit, for discussion Suppose $d_1, d_2, d_3, \ldots$ is a sequence such that $\lim_{n\to\infty} d_n = L$ where L is finite. Let $f_n = (\sum_{i=1}^{n} d_i)/n$. Show that $\lim_{n\to\infty} f_n = L$ as well.

Q4 Prove that if $\lim_{n\to\infty} c_n = L$ for some $L \in \mathbb{R}$ then $\lim_{n\to\infty}(c_n)^2 = L^2$. On the other hand, give an example of a sequence c_n and a real number L where $\lim_{n\to\infty}(c_n)^2 = L^2$ but where it is not true that $\lim_{n\to\infty} c_n = L$.

Q5 Let a_n, $n \geq 1$ be the sequence defined recursively by $a_1 = 1$, $a_{n+1} = (a_n + 2/a_n)/2$. Prove that the sequence converges to the limit $\sqrt{2}$.