Math 512- Applied Nonstandard Analysis
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While the use of infinitesimal and infinite elements was prevalent in the early development of calculus and real analysis and was used by preeminent mathematicians such as Leibniz and Cauchy, the dubious nature of such elements forced the need for the now common ϵ-δ rigor. In the 1960s, using ideas from model theory, Abraham Robinson rescued the infinitesimals from their squalid state and gave them a firm, foundational background; thus was the birth of the area of mathematics now (unfortunately) known as *nonstandard analysis*.

In the years since its inception, nonstandard analysis has been used to prove theorems in virtually all areas of mathematics. Besides offering simpler heuristics for notions that often seem more complicated to express in standard terms, nonstandard analysis also offers new proof techniques, including the notions of *overflow/underflow* and *saturation*.

In this course, we will introduce nonstandard methods in a way that should be understandable by anybody with a modest background in mathematics; in particular, no knowledge of logic will be assumed. After setting up the framework, we will warm up by studying some elementary real analysis from the nonstandard point of view.

Once we are familiar with the nonstandard way of reasoning, we will work our way towards two main goals: the nonstandard solution of Hilbert's fifth problem due to Hirschfeld and the recent solution to the classification of approximate groups due to Breuillard, Green, and Tao (building on ideas of Hrushovski and worked out a bit further by van den Dries). The latter work implies, as a simple corollary, a strengthening of a theorem of Gromov stating that a group of polynomial growth is virtually nilpotent.

If time permits, we might also look at some recent applications of nonstandard methods in combinatorial number theory.