Homework 3: Set Theory, Fall 1998
Due October 19

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1. The Hessenberg sum of ordinals is defined as follows:
If *alpha* and α' are written in Cantor normal form as

$$\alpha = \sum_{i<k} \omega^{\beta_i} \cdot m_i$$

and

$$\alpha' = \sum_{i<k} \omega^{\beta_i'} \cdot m_i'$$

(I am allowing some m_i to be zero to save typing.) then

$$\alpha \oplus \alpha' = \sum_{i<k} \omega^{\beta_i'} \cdot (m_i + m_i').$$

Now show:

1. $\oplus$ agrees with $+$ if α and β are finite.
2. $\oplus$ is commutative.
3. How do you determine which of two ordinals written in Cantor normal form is larger?
4. If $\beta < \beta'$ then $\alpha \oplus \beta < \alpha \oplus \beta'$.
5. Show the following further form of 'indecomposable'. If $\alpha = \omega^\delta$ some δ then for any $\beta, \gamma < \alpha$, $\beta \oplus \gamma < \alpha$.

2. Show that the Hercules slays the hydra no matter what strategy he follows. (Hint: Cantor normal form is key.)

problems 14,15, 18, 19 (cardinals).

Feel free to e-mail me at jbaldwin@uic.edu or phone to make an appointment to discuss any difficulties that arise. Office: 417 SEO. Office phone: 312-413-2149