Model Theory, Spring 2001
Problem Set 2: due Feb. 12

Marker: 2.5.17 (in your notes); 3.4.12, 3.4.13, 3.4.14 being handed out.

1. Let $p \in S(A)$ be an algebraic type. I.e there are only finitely many realizations of p. Show p is principal (i.e. isolated). More generally, show that if $\mathrm{tp}(\mathrm{a_1}, \ldots \mathrm{a_n}/\mathrm{A})$ is algebraic then each $a_i \in \mathrm{acl}(A)$.

2. Let T be an uncountably categorical theory. Show every uncountable model of T is $\aleph_0$-saturated. (This should be very easy assuming Morley's theorem. How much of it can you get from the preliminary results?)

3. Show that if T is $\aleph_1$-but not $\aleph_0$-categorical, the prime model of T is minimal – has *no* proper elementary submodel.

4. Let T be any complete countable theory. Prove that for any cardinal κ, there is a model M of T with cardinality κ such that every definable subset of M is either finite or has cardinality κ.

5. T' is said to be an inessential extension of T of T' is the theory of $(M, x)_{x \in X}$ where X is a subset of M. Show that if $|X| < \kappa$ and T is κ-categorical then T' is κ-categorical.

6. (Uncountable languages). Show that if T is the theory of vector spaces over an uncountable field F then T is $|F|^+$-categorical but has a model of cardinality $|F|$ which is not homogeneous. (This contrasts with a result we may prove: every countable model of an $\aleph_1$-categorical countable theory is homogenous– indeed $\aleph_1$-homogenous.)