Model Theory, Spring 2001
Problem Set 3: due Mar. 2/Mar 16

Be prepared to discuss several of these questions and explain how you have begun them in class on March 2. The extra time will be work them out; there may be a few added.

1. Let $p \in S(B)$ be an algebraic type. Suppose B is not contained in the algebraic closure of $A \subseteq B$. Prove p forks over A.

2. Let $p \in S(B)$ and suppose B is contained in the algebraic closure of $A \subseteq B$. Prove p does not fork over A. (This may get clearer after a few more lectures.)

3. $\phi(\overline{x}, \overline{y})$ has the order property if there exist $\overline{a}_i, \overline{b}_i$ for $i < \omega$ such that $\phi(\overline{a}_i, \overline{b}_j)$ iff $i < j$. Prove that if ϕ has the order property then T is unstable. Think about the converse; if you really want to prove it, you will need a combinatorial lemma that can be found in the appendix to Shelah. Try to conjecture the lemma.

4. M is a conservative extension of N if for every $\overline{m} \in M$, and every $\phi(\overline{x}, \overline{y})$ the subset of N (to an appropriate power): $\phi(M, \overline{m}) \cap N$ is also given by $\psi(\overline{x}, \overline{m}^*)$ for an appropriate ψ and $\overline{m}^* \in N$. Show T is stable iff every elementary extension of a model of T is a conservative extension.

5. Show that if d mapping $\phi(\overline{x}, \overline{y})$ - an $L(T)$ formula to $d\phi(\overline{y}, \overline{m})$ which defines $t(\overline{c}/M)$ is a Boolean homomorphism from the Boolean algebra of $L(T)$-formulas to the Boolean algebra of $L(T)$-formulas with parameters from M. Find an example to show it is essential that M be a model.

6. Show that $p = t(\overline{a}/M\overline{c})$ is a coheir of $t(\overline{a}/M)$ iff p is in the topological closure of $\{t(\overline{m}/M\overline{c}); \overline{m} \in M\}$. ($\overline{m}$ and $\overline{a}$ have the same length.)

Definition: The theory T does not have the *finite cover property* if for every finite set of formulas Δ, there is a k such that a Δ-type p is consistent if every k-element subset of it is consistent.

7. Show that if T is stable without the finite cover property there for every formula $\phi(\overline{x}, \overline{y})$ and every A with $|A| \geq 2$ there is a pair of formulas: $(d\phi(\overline{y}, \overline{z}), \chi(\overline{z}))$ such that:

1. If $\chi(\overline{c})$, $\phi(\overline{x}, \overline{y})$ iff $d\phi(\overline{y}, \overline{c})$ defines a complete consistent type over A.
2. for every A and $p \in S(A)$, there is $\overline{c}_p \in A$ with $\chi(\overline{c}_p)$ and $d\phi(\overline{c}_p)$ defines p.

(I will hand out copies of 4-5 pages from my book which gives the background on definability.

(Aside: If T does not have the finite cover property then T is stable but I didn't want to confuse this argument with that additional fact.)