Model Theory, Spring 2001
Problem Set 4: due Apr. 9

Be prepared to discuss several of these questions and explain how you have begun them in class on Apr 2. The extra time will be work them out; there may be a few added.

A theory is *small* if there are only countably many n-types over the empty set (for every n).

1. Show that a small countable theory T is ω stable if and only if for every $M \models T$ and $p \in S(M)$, p is definable over a finite subset of M.

2. If T is stable, E is a set of indiscernibles over A with $|E| \geq \overline{\kappa}(T)$, there is an $E_0 \subseteq E$ such that $|E_0| < \overline{\kappa}(T)$, $E - E_0$ is independent over E_0 and all elements of $E - E_0$ realize the same strong type over E_0.

3. Show that if T is a small superstable countable theory and I is a countable set of indiscernibles over $\emptyset$ then $|S(I)| = \aleph_0$.

4. Show that if α is an automorphism fixing A then for any a, b, if $stp(a/A) = stp(b/A)$ then $stp(\alpha a/A) = stp(\alpha b/A)$. Give an example where $stp(a/A) \neq stp(\alpha a/A)$.

The type p is almost satisfied in A if every finite subset of p is realized in every model containing A.

5. (T stable) Show p does not fork over A iff p is almost satisfied in A. What goes wrong for simple T?