MATH 121 SAMPLE FINAL PARTIAL SOLUTIONS 12/4/1996
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1. Set $f(x) = x^4 + x^3 - 2x^2 - 4x - 8$. The rational solutions to $f(x) = 0$ are among $\pm 1, \pm 2, \pm 4, \pm 8$. That there are 2 rational solutions is given. Graphing $y = f(x)$ yields that ± 2 are the two. Thus $x - 2, x + 2$ divide $f(x)$; hence $x^2 - 4$ divides $f(x)$. Long division gives $f(x) = (x^2 - 4)(x^2 + x + 2)$. The roots of x^2+x+2 are $-\frac{1}{2} \pm \frac{\sqrt{7}}{2}i$. Thus $2, -2, -\frac{1}{2} + \frac{\sqrt{7}}{2}i, -\frac{1}{2} - \frac{\sqrt{7}}{2}i$ are the roots. Followup question: What is the factorization of $f(x)$ into linear factors?

2. $P(x) = a(x - (-4))(x - 3)^2 = a(x + 4)(x - 3)^2$ since $P(x)$ has degree 3, $x = -4$ and $x = 3$ are roots of multiplicity 1 and 2 respectively. Implicit is that the coefficients of $P(x)$ are real numbers. Since $P(x) \longrightarrow +\infty$ as $x \longrightarrow -\infty$ we conclude that $a < 0$. (a) .... (e) $P(x)$ has roots $x = -4, x = 3$. Since $P(-100) > 0, P(0) < 0, P(4) < 0$, it follows that $P(x) < 0$ on $(-4, 3) \cup (3, \infty)$. The other parts are straightforward.

3. (a) $\ln(x + 1) + \ln(x + 7) = \ln(x + 2) + \ln(x + 3)$ means $\ln(x + 1)(x + 7) = \ln(x+2)(x+3)$. Since ln is one-one it follows that $(x+1)(x+7) = (x+2)(x+3)$. Expanding both sides we have $x^2 + 8x + 7 = x^2 + 5x + 6$ and thus $3x = -1$. Therefore $x = -\frac{1}{3}$. *Check to see that this is in fact a solution by substituting into the original equation.*

(b) $\log_a c = \log_b c/\log_b a$, so $\log_7 13 = \frac{\ln 13}{\ln 7}$. Work out the exact value to 4 decimal places.

4. Let $z = e^x$ Then $\frac{1}{z} = e^{-x} \neq 0$ since the range of an exponential function consists of positive numbers. The equation $e^x - e^{-x} = 6$ is thus $z - \frac{1}{z} = 6$. Therefore, $z^2 - 1 = 6z$ so $z^2 - 6z - 1 = 0$. By the quadratic formula $e^x = z = \frac{6 \pm \sqrt{40}}{2}$. Therefore $x = \ln(\frac{6 \pm \sqrt{40}}{2})$. Note $\frac{6 - \sqrt{40}}{2} < 0$. Thus there is one answer, $x = \ln(\frac{6+\sqrt{40}}{2})$. Work out the answers to 4 decimal places.

Since $2^{3x+1} = 3^{2x+1}$, $(3x + 1)\ln 2 = (2x + 1)\ln 3$. Therefore $(3\ln 2 - 2\ln 3)x = \ln 3 - \ln 2$ or $(\ln 8 - \ln 9)x = \ln 3 - \ln 2$. Thus $(\ln \frac{8}{9})x = \ln \frac{3}{2}$ and $x = \frac{\ln \frac{3}{2}}{\ln \frac{8}{9}}$. (Work out the answer to 4 decimal places.)

5. $Q(t) = Q(0)e^{kt}$, so $5Q(0) = Q(5.83) = Q(0)e^{5.83k}$ means that $5 = e^{5.83k}$. Therefore $\ln 5 = 5.83k$ and hence $k = \frac{\ln 5}{5.83}$. At the time to triple in size $Q(t) = 3Q(0)$. Thus $3Q(0) = Q(t) = Q(0)e^{kt}$ means $3 = e^{kt}$ and therefore $t = \frac{\ln 3}{k} = \frac{\ln 3}{(\frac{\ln 5}{5.83})} = \frac{5.83 \ln 3}{\ln 5}$. $Q(1) = Q(0)e^k$ so $\frac{Q(1)}{Q(0)} = e^k$. (Work out the explicit answers. Note that if $P(t) = Pa^{kt}$ then $P(0) = P$.)

6. The slope of the line passing L thru points (a_1, b_1) and (a_2, b_2), when $a_1 \neq a_2$, is $m_L = \frac{b_2 - b_1}{a_2 - a_1}$. For points $(7, 8)$ and $(8, 7)$ we compute $m_L = \frac{7-8}{8-7} = -1$. Thus $y - 7 = (-1)(x - 8)$ is an equation of the line (point slope form). The *slope-intercept form* is $y = -x + 15$.

Let L_{perp} be the line perpendicular to the line L determined by $2x - 5y = 10$, and suppose L_{perp} passes thru $(2, 2)$. The slope-intercept form of L is $y = \frac{2}{5}x - 2$. Thus $m_L = \frac{2}{5}$. Since $m_L m_{L_{perp}} = -1$ it follows that $m_{L_{perp}} = -\frac{5}{2}$. Thus L_{perp} has equation $y - 2 = -\frac{5}{2}(x - 2)$. The *slope-intercept form* of the equation of L_{perp} is thus $y = -\frac{5}{2}x + 7$. 7. $f(x) = 3 - \sqrt{x^3 - 8}$. Thus the domain of $f(x)$ consists of all numbers x such that $x^3 - 8 \geq 0$, or $x^3 \geq 8$, or $x \geq 2$ since $y = x^3$ is an increasing function. The *domain of* $f(x)$ is $[2, \infty)$. To find the range of $y = f(x)$ note that $\sqrt{x^3 - 8} \geq 0$, so $y = 3 - \sqrt{x^3 - 8} \leq 3 - 0 = 3$. If $y \leq 3$ then $y = 3 - \sqrt{x^3 - 8}$ has solution $x = ((3 - y)^2 + 8)^{\frac{1}{3}}$. Therefore the *Range* of $f(x)$ is $(-\infty, 3]$. Note that $y = f(x)$ is one-one. Thus $f(x)$ has an inverse, and we have shown that $f^{-1}(x) = ((3-x)^2+8)^{\frac{1}{3}}$. *Domain* $f(x) =$ *Range* $f^{-1}(x)$ and thus *Range* $f(x) =$ *Domain* $f^{-1}(x)$ hold generally for a function with an inverse. Thus $[2, \infty) = Domain f(x) = Range f^{-1}(x)$ and $Domain f^{-1}(x) = Range f(x) = (-\infty, 3]$.

Let $y = g(x) = \frac{2x-1}{3x+2}$. To find $g^{-1}(x)$ we set $x = \frac{2y-1}{3y+2}$ and solve for y in terms of x: $(3y + 2)x = 2y - 1, (3x - 2)y = -1 - 2x$ and hence $y = \frac{-1-2x}{3x-2} = \frac{1+2x}{2-3x}$. Therefore $g^{-1}(x) = \frac{1+2x}{2-3x}$. $\{x \mid x \neq -\frac{2}{3}\} =$ Domain $g(x) =$ Range $g^{-1}(x)$ and $\{x \mid x \neq \frac{2}{3}\} =$ Domain $g^{-1}(x) =$ Range $g(x)$.

8. $(f \circ g)(3) = f(g(3)) = f(\frac{3+2}{3-2}) = f(5) = \sqrt{19 + 5^3} = \sqrt{144} = 12$. $(g \circ f)(5) = g(f(5)) = g(12) = \frac{12+2}{12-2} = \frac{14}{10}$.

9. Solve $1 - 4x - x^2 = |2x - 3|$. Case 1: $2x - 3 \geq 0$. The equation becomes $1 - 4x - x^2 = 2x - 3$, or equivalently $x^2 + 6x - 4 = 0$. Thus $x = \frac{-6 \pm \sqrt{52}}{2} = -3 \pm \sqrt{13}$ by the quadratic formula. Since $2x - 3 \geq 0$, necessarily $x = -3 + \sqrt{13}$. In this case $2x - 3 = -6 + 2\sqrt{13} - 3 = 2\sqrt{13} - 9 < 0$. No solution from this case. Case 2: $2x - 3 < 0$. The equation in this case is $1 - 4x - x^2 = -(2x - 3)$ or equivalently $x^2 + 2x + 2 = 0$. By the quadratic formula there are no real roots. Conclusion: There are *no* real solutions to $1 - 4x - x^2 = |2x - 3|$.

Consider $|2x + 1| = |x|$. Then $\pm(2x + 1) = \pm x$ as $|x| = \pm x$, so we solve $2x + 1 = \pm x$. If $2x + 1 = x$ then $x = -1$. If $2x + 1 = -x$ then $x = -\frac{1}{3}$. *Substituting these values into the original equation, we see they are* *both solutions.*

10. $y = 20e^{Kx}$. If $(1, 30)$ is a point of the graph then $30 = 20e^{K1} = 20e^{k}$, so $\frac{30}{20} = e^K$ and $\ln\frac{3}{2} = K$. note $y(0) = 20$ is the initial value of y. Thus when y has doubled, $40 = y = 20e^{Kx}$ so $2 = e^{Kx}$ and $\ln 2 = Kx$. Therefore $x = \frac{\ln 2}{K} = \frac{\ln 2}{\ln\frac{3}{2}}$.

If the graph of $y = 20e^{Kx}$ has the point $(30, 1)$ then $1 = 20e^{K30}$ or $\frac{1}{20} = e^{K30}$. Therefore $\ln\frac{1}{20} = 30K$ or $\frac{-\ln 20}{30} = K$.

11. $P(t) = P(1+\frac{r}{k})^{kt}$ is the return on an initial investment of P after t years with an annual interest rate of $100r\%$ compounded k times annually. Comparing investment schemes we need to solve $P(1+\frac{r}{12})^{12} = P(1+.0575)$ as $t = 1$. Thus $(1+\frac{r}{12})^{12} = 1+.0575$ and $12\ln(1+\frac{r}{12}) = \ln 1.0575$. Therefore $\ln(1+\frac{r}{12}) = \frac{\ln 1.0575}{12}$ so $1+\frac{r}{12} = e^{\frac{\ln 1.0575}{12}}$ and $r = 12(e^{\frac{\ln 1.0575}{12}} - 1)$. The annual percentage rate is $100r\%$.

Compounding continuously is given by $P(t) = Pe^{rt}$. In this comparison we have $Pe^r = P(1+.0575)$ and hence $e^r = 1.0575$. Therefore $r = \ln 1.0575$.

Assume that we compound continuously at an annual rate of 100%, starting with \$1. After 1 year we have $P(1) = 1e^{r1} = e$ dollars as $r = 1$ in this case.

12. $f(x) = \frac{x^2-5x+4}{(x+1)(x^2-3)} = \frac{(x-1)(x-4)}{(x+1)(x+\sqrt{3})(x-\sqrt{3})}$. To solve $f(x) < 0$ we compute the real roots of the denominator and numerator of $f(x)$ and use these as endpoints of intervals on the real number line. These roots are (in order) $-\sqrt{3}, -1, 1, \sqrt{3}, 4$. $f(x)$ will be positive or negative on each of these intervals. The calculations $f(-10) < 0, f(-1.5) > 0, f(0) < 0, f(1.5) > 0, f(3) < 0, f(10) > 0$ show that $f(x) < 0$ on $(-\infty, -\sqrt{3}) \cup (-1, 1) \cup (\sqrt{3}, 4)$.

Solve $g(x) = x + \frac{1}{x} \geq 0$. Write $g(x) = \frac{x^2+1}{x}$. The real roots of the numerator and denominator of $g(x)$ are 0. As $g(-1) < 0$ and $g(1) > 0$ we conclude that $g(x) > 0$ on $(0, \infty)$. Since $x = 0$ is not in the domain of $g(x)$ the solutions to $g(x) \geq 0$ consist of $(0, \infty)$.

13. The most general form of the function is $f(x) = \frac{ax^2+bx+c}{(x-1)(x+1)(x-5)} + x^3 - x$. (Generally if $f(x) = \frac{p(x)}{q(x)}$ and $p(x) = Q(x)q(x) + R(x)$, where $R(x) = 0$ or $\mathrm{Deg}R(x) < \mathrm{Deg}q(x)$, then $f(x) = \frac{p(x)}{q(x)} = Q(x) + \frac{R(x)}{q(x)}$ has end behaviour asymptote $y = Q(x)$.) Taking $a = b = 0$ and $c = 1$ we have $f(x) = \frac{1}{x^3-5x^2-x+5} + x^3 - x = \frac{1+(x^3-5x^2-x+5)(x^3-x)}{x^3-5x^2-x+5} = \frac{x^6-5x^5-2x^4+10x^3+x^2-5x+1}{x^3-5x^2-x+5}$. Replace $x^3 - x$ by $x - x^2$ and redo the previous calculation.

14. Recall that $\sin(x+\frac{\pi}{2}) = \cos x$ and $\cos(x-\frac{\pi}{2}) = \sin x$. Also *function* $(\frac{\pi}{2}-x) = $ *cofunction*x; thus $\sin(\frac{\pi}{2}-x) = \cos x$, $\cos(\frac{\pi}{2}-x) = \sin x$, $\cot(\frac{\pi}{2}-x) = \tan x$, etc. Recall that $\sin(-x) = -\sin x$ and $\cos(-x) = \cos x$; that is sin is an *odd* function and cos is an *even* function. Thus $\sin(x-\pi) = \sin((x-\pi)+2\pi) = \sin(x+\pi) = \sin((x+\frac{\pi}{2})+\frac{\pi}{2}) = \cos(x+\frac{\pi}{2}) = \cos(-x-\frac{\pi}{2}) = \sin(-x) = -\sin x$. Also, $\sin(x+\frac{3\pi}{2}) = \sin((x+\frac{3\pi}{2})-2\pi) = \sin(x-\frac{\pi}{2}) = -\sin(\frac{\pi}{2}-x) = -\cos x$. Try to derive these identities, and the others, by looking at the unit circle.

15. $\cos(\frac{\pi}{8}) = \cos(\frac{1}{2}(\frac{\pi}{4})) = \sqrt{\frac{1+\cos\frac{\pi}{4}}{2}} = \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2+\sqrt{2}}{4}} = \frac{\sqrt{2+\sqrt{2}}}{2}$. $\sin(\frac{\pi}{12}) = \sin(\frac{\pi}{3}-\frac{\pi}{4}) = \sin(\frac{\pi}{3})\cos(\frac{\pi}{4})-\cos(\frac{\pi}{3})\sin(\frac{\pi}{4}) = (\frac{\sqrt{3}}{2})(\frac{\sqrt{2}}{2})-(\frac{1}{2})(\frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{4}(\sqrt{3}-1)$. Note that $\cos 75^o = \cos(45^o+30^o) = \cos 45^o \cos 30^o - \sin 45^o \sin 30^o$..... , and $\cos 165^o = \cos(180^o - 15^o) = -\cos 15^o = -\cos(\frac{1}{2}(30^o))$.

16. $y = \cot x = \tan(\frac{\pi}{2}-x) = -\tan(x-\frac{\pi}{2})$. See problem 14 above. The last equation follows since $y = \tan x$ is an *odd* function. Since $\cot = -\tan(x-\frac{\pi}{2})$ we can describe the graph of $y = \cot x$ as derived from the graph of $y = \tan x$ by (1) a horizontal shift right of $\frac{\pi}{2}$ and (2) reflection through the x-axis. Note that a is a zero of $y = \tan x$ if and only if $x = a$ is a vertical asymptote of $y = \cot x$, and a is zero of $y = \cot x$ if and only if $y = a$ is a vertical asymptote of $y = \tan x$.

$y = \cos x = \sin(x+\frac{\pi}{2})$, so the graph of $y = \cos x$ is obtained from the graph of $y = \sin x$ by a horizontal shift to the left $\frac{\pi}{2}$.

17. $\tan(\cos^{-1}\frac{x}{8}) = \frac{\sin(\cos^{-1}\frac{x}{8})}{\cos(\cos^{-1}\frac{x}{8})} = \pm\frac{\sqrt{1-\cos^2(\cos^{-1}\frac{x}{8})}}{\frac{x}{8}} = \pm\frac{\sqrt{1-\frac{x^2}{64}}}{\frac{x}{8}}$. Show that the domain of this composite is $[-8,0)\cup(0,8]$. Show that if $x \in [-8,0)\cup(0,8]$ then $\tan(\cos^{-1}\frac{x}{8}) = \frac{\sqrt{64-x^2}}{x}$. Also solve this problem by constructing a right triangle with hypoteneus of length 8 and adjacent side of length x.

18. $\sin(3x) = \sin(2x+x) = \sin 2x \cos x + \cos 2x \sin x = (2\sin x\cos x)(\cos x) + (1-2\sin^2 x)\sin x = 2\sin x(1-\sin^2 x) + (1-2\sin^2 x)\sin x = 3\sin x - 4\sin^3 x$. Thus $\sin 30^o = 3\sin 10^o - 4\sin^3 10^o$, so as $\sin 30^o = \frac{1}{2}$ we deduce that $4\sin^3 10^o - 3\sin 10^o + \frac{1}{2} = 0$.

19. By the *Law of Sines* $\frac{\sin 34^o}{6} = \frac{\sin\alpha}{7}$. Therefore $\sin\alpha = \frac{7}{6}\sin 34^o$. *If* $-\frac{\pi}{2} \le \alpha \le \frac{\pi}{2}$ *then* $\alpha = \sin^{-1}(\frac{7}{6}\sin 34^o) \approx 40.72^o$. In any case the (approximate) possibilities for α are 40.72^o and $180^o - 40.72^o = 139.28^o$. Since $h = 6\sin 34^o \approx 3.55$ both are realized.

Case 1: $\alpha \approx 40.72^o$. Then $\gamma \approx 180^o - (34+40.72)^o = 105.28^o$. Therefore $\frac{\sin 105.28^o}{c} \approx \frac{\sin 34^o}{6}$ and consequently $c \approx 6(\frac{\sin 105.28^o}{\sin 34^o}) \approx 10.35$. Check by comparing the values $\frac{\sin 34^o}{6}, \frac{\sin 40.72^o}{7}, \frac{\sin 105.28^o}{10.35}$ which should be about the same by the law of sines. In this case the area is given by $Area = \frac{1}{2}ac\sin\beta = \frac{1}{2}(7)(10.35)\sin 34^o \approx 20.26$.

Case 2: $\alpha \approx 139.28^o$. Check that $\gamma \approx 6.72^o, c \approx 1.26$.

(Note: In neither case are all of the angles acute.)

20. By the *Law of Cosines* we have $5^2 = 7^2 + 8^2 - 2(7)(8)\cos\alpha$. Thus $25 = 47 + 64 - 112\cos\alpha$, or $112\cos\alpha = 113 - 25 = 88$. Therefore $\cos\alpha = \frac{88}{112} = \frac{11}{14}$. The range of $\cos^{-1}$ is $[0,\pi]$. Thus $\alpha = \cos^{-1}\frac{11}{14} \approx 38.21^o$, or about .67 radians.

To find the area we first compute the semiperimeter $s = \frac{1}{2}(a+b+c) = \frac{1}{2}(5+7+8) = 10$ and use the formula $Area = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{10(5)(3)(2)} = 10\sqrt{3}$.

21. *linear velocity = radius* $\times$ *angular velocity*, or $v = r\omega$. 40 miles per hour = (40)(5280) feet per hour = $\frac{(40)(5280)}{(60)(60)}$ feet per second. Thus $\frac{(40)(5280)}{(60)(60)} = \frac{2}{3}\omega$ as 8 inches $= \frac{2}{3}$ feet. Thus $\omega = \frac{(3)(40)(5280)}{(2)(60)(60)} = 88$ radians per second. This translates to $\frac{\omega}{2\pi} \approx 14.01$ revolutions per second, or $(\frac{\omega}{2\pi})(60) \approx 840.34$ revolutions per minute.

When the angular speed is $\omega = 2\pi$ radians per second, the linear speed is $r\omega = (\frac{2}{3})(2\pi) \approx 4.1888$ feet per second, or $(\frac{2}{3})(2\pi)(\frac{(60)(60)}{5280}) \approx 2.8560$ miles per hour.

60 miles per hour $= (60)(\frac{5280}{60}) = 5280$ feet per minute. So $5280 = \frac{2}{3}\omega$ means that $\omega = (\frac{3}{2})(5280) = 7920$ radians per minute which gives $\frac{\omega}{2\pi} = \frac{7920}{2\pi} \approx 1260.51$ revolutions per minute (rpm).

22. Write $y = f(x) = a\sin(bx+c) + d$. *We may assume that* $a, b > 0$. Note that $y_{max} = a + d$ and $y_{min} = -a + d$. Therefore $d = \frac{1}{2}(y_{max} + y_{min})$ and $a = \frac{1}{2}(y_{max} - y_{min})$. Since $y_{max} = 4.8$ and $y_{min} = 2.0$ we have $d = \frac{1}{2}(4.8 + 2.0) = \frac{1}{2}(6.8) = 3.4$ and $a = \frac{1}{2}(4.8 - 2.0) = \frac{1}{2}(2.8) = 1.4$. Thus $y = 1.4\sin(bx+c) + 3.4$. Now *if a maximum occurs at* $x = x_1$ *and the next minimum occurs at* $x = x_2$ *then* $x_2 - x_1$ *is half a period.* Therefore the period is $2(8.8) = 17.6$. Since $|b|(period) = 2\pi$, we conclude that $|b| = \frac{2\pi}{17.6}$. Thus $y = 1.4\sin((\frac{2\pi}{17.6})x + c) + 3.4 = 1.4\cos((\frac{2\pi}{17.6})x + c - \frac{\pi}{2}) + 3.4$. A specific function which is consistent with the given requirements is $y = f(x) =$

$1.4\sin((\frac{2\pi}{17.6})x)+3.4 = 1.4\cos((\frac{2\pi}{17.6})x-\frac{\pi}{2})+3.4 = 1.4\cos((\frac{2\pi}{17.6})(x-\frac{17.6}{4}))+3.4.$

23. $z = -1 + i\sqrt{3} = a + ib$ where $a = -1$ and $b = \sqrt{3}$. Thus $r = |z| = \sqrt{a^2 + b^2} = \sqrt{(-1)^2 + (\sqrt{3})^2} = 2$. Writing $z = r(\cos\theta + i\sin\theta) = 2(-\frac{1}{2} + i\frac{\sqrt{3}}{2})$ we have that $\cos\theta = -\frac{1}{2}$ and $\sin\theta = \frac{\sqrt{3}}{2}$. The reference angle is $\frac{\pi}{3}$ and we may take $\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$. Thus $z^6 = r^6(\cos 6\theta + i\sin 6\theta) = 2^6(\cos 6(\frac{2\pi}{3}) + i\sin 6(\frac{2\pi}{3}) = 64(\cos 4\pi + i\sin 4\pi) = 64 + i0 = 64$.

Let $\omega = -2-2i$. Then $|\omega| = 2\sqrt{2}$ so $\omega = 2\sqrt{2}(-\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}) = 2\sqrt{2}(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4})$. Therefore $\omega^5 = (2\sqrt{2})^5(\cos 5(\frac{5\pi}{4}) + i\sin 5(\frac{5\pi}{4})) = 2^5(2)^{\frac{5}{2}}(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}) = 2^7\sqrt{2}(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i) = 2^7(1+i)$ since $\frac{25\pi}{4} = 6\pi + \frac{\pi}{4}$. Note that $z^{10} = 2^{10}(\cos 10(\frac{2\pi}{3}) + i\sin 10(\frac{2\pi}{3})) = 2^{10}(\cos\frac{20\pi}{3} + i\sin\frac{20\pi}{3}) = 2^{10}(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}) = 2^{10}(-\frac{1}{2} + i\frac{\sqrt{3}}{2}) = 2^9(-1 + i\sqrt{3})$. Thus $\omega^5/z^{10} = \frac{2^7\sqrt{2}}{2^{10}}(\cos(\frac{\pi}{4} - \frac{2\pi}{3}) + i\sin(\frac{\pi}{4} - \frac{2\pi}{3})) = \frac{\sqrt{2}}{8}(\cos(-\frac{5\pi}{12}) + i\sin(-\frac{5\pi}{12})) = \frac{\sqrt{2}}{8}(\cos\frac{5\pi}{12} - i\sin\frac{5\pi}{12})$. Noting $\frac{5\pi}{12} = \frac{\pi}{6} + \frac{\pi}{4}$ complete this calculation. We will compute ω^5/z^{10} another way. Note that $\omega^5/z^{10} = \frac{2^7(1+i)}{2^9(-1+\sqrt{3})} = \frac{1}{4}(\frac{1+i}{-1+\sqrt{3}}) = \frac{1}{4}(\frac{1+i}{-1+i\sqrt{3}})(\frac{-1-i\sqrt{3}}{-1-i\sqrt{3})}) = \frac{1}{4}(\frac{-1+\sqrt{3}}{4} + \frac{-1-\sqrt{3}}{4}i) = \frac{-1+\sqrt{3}}{16} - \frac{1+\sqrt{3}}{16}i$.

24. $-32i = 32(0 - i) = 32(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2})$. Thus the 5^{th} roots of z are $\omega = 2(\cos(\frac{\frac{3\pi}{2}+2\pi k}{5}) + i\sin(\frac{\frac{3\pi}{2}+2\pi k}{5})) = 2(\cos(\frac{3\pi+4\pi k}{10}) + i\sin(\frac{3\pi+4\pi k}{10}))$ for $k = 0, 1, 2, 3, 4$. For $k = 2$ for example, $\omega = 2(\cos\frac{11\pi}{10} + i\sin\frac{11\pi}{10})$ which needs to be worked out with a calculator.

$1 = 1 + 0i = 1(\cos 0 + i\sin 0)$. Therefore the 8^{th} roots of unity are $\omega = 1(\cos\frac{0+2\pi k}{8} + i\sin\frac{0+2\pi k}{8}) = \cos\frac{\pi k}{4} + i\sin\frac{\pi k}{4}$ for $k = 0, 1, 2, 3, 4, 5, 6, 7$. See the text for the rest of the details of the calculation.