MATH 121 FINAL EXAM FALL 1996

Name:______________________________ SSN:____________________

The exam consists of 15 problems. Each problem is worth 12 points except for 2 (16 points), 5 (20 points) and 11 (20 points).

All work must be shown to earn credit for any problem.

Write the name of your Lecturer at the top of your answer booklet. Also include your name, the name of your TA, and the time and room of your TuTh discussion section on the front of your answer booklet.

Place your exam in your answer booklet before turning it in.

1. Find the equation in slope-intercept form of the line passing through the point $(1, 2)$ and parallel to the line $x + 4y = 1$.
   Answer: $y = -x - 9/4$.
   3 points for finding slope of given line.
   6 points for setting up equation of new line with appropriate substitutions.
   3 points for converting new equation to slope-intercept form.

2. Let $f(x) = 9 + \sqrt{x-5}$.
   a) Determine the domain and range of the function f.
   b) Find (a formula for) the inverse function f^{-1} and determine the domain of f^{-1}.
   Answer: a) range $[9, \infty)$, domain $[5, \infty)$.
   b) range: $[5, \infty)$, domain $[9, \infty)$
   6 points for each part; deduct 1 point if open-closed confused (maximal deduction 4); deduct 1 point for a description that is intelligible but not mathematically correct, e.g. $x \geq 9$ instead of "all $x \geq 9$ or $\{x : x \geq 9\}$ or $[9, \infty)$ (maximal deduction 2).

3. Find all solutions of $2 - 5x = |x - 10|$.
   Answer: x = -2.
   Two methods of solution: graphically: we didn't specify should
   accept within .1.
   algebraically: 6 points if student answers 2, -2.
   Answer:

4. The equation $x^3 + 4x^2 + 8x + 8 = 0$ has a rational root. Find all complex solutions of this equation (including, of course, all real solutions). Give exact values of the solutions with complex numbers written in standard form ($a + bi$, a, b real) with their real and imaginary parts expressed as simply as possible.
   Answer: -2, $-1 \pm i$.
   Here we required exact; no credit for graphical solution with approximate roots.

5. Let $P(x)$ be a polynomial of degree 3 that has a root of multiplicity 1 at $x = -1$ and a root of multiplicity 2 at $x = 1$. Assume that $P(x) \to +\infty$ as $x \to -\infty$.

   a) What does $P(x)$ approach as $x \to +\infty$?

   b) Is $P(-2)$ positive or negative?

   c) What is the solution set for the inequality $P(x) < 0$?

   Express your answer in interval notation as an interval or the union of intervals.

   d What does $1/P(x)$ approach as $x \to 1^-$?

   Answer: a) $-\infty$, b) positive, c) $(-\infty, -1)$, d) $-\infty$.

   3 points for each. in c) we this time demanded interval notation.

6. Consider a quantity $Q(t)$ that grows exponentially according to the law $Q(t) = Q(0)e^{kt}$, where t is the time in hours and k is a constant. If $Q(t)$ triples in size in 35.28 hours, how long does it take for $Q(t)$ to double in size. Give your answer in hours, correct to 2 decimal places.

   answer: 22.51 days (k = .0307)

   6 points for determining constant; 6 points from determining doubling time from whatever constant was determined.

7. What simple annual rate of interest yields the same return for an account as an annual rate of 9.5% compounded monthly? Give your answer in per cent form, correct to 2 decimal places.

   answer: $(1 + (.095/12)^{12} = 1.0992$, so 9.92%

   I don't see any partial credit except for obvious miscopying or rounding errors (1 or 2 points off)

8. Solve $\dfrac{x^2 - 7}{x^2 - x - 6} < 0$.

   Answer: $(-\sqrt{7}, -2) \cup (\sqrt{7}, 3)$.

   We didn't specify exactly; a graphical solution correct to 2 places should be accepted (with picture)."

9. Write a rational function $f(x)$ that has vertical asymptotes $x = -3$, $x = 2$ and that has the end behavior asymptote $g(x) = 2x + 1$.

   Answer: $3x - 1 + r/(x + 3)(x - 2)$ for any real r.

   8 points for indentifying the denominator; 4 for the asymptote.

10. Use the angle sum formula $\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$ to express the following in terms of $\sin(x)$ only:

    a) $\cos(x + \pi/2)$.

    b) $\cos(x - \pi/2)$.

    Answer: a) $-\sin x$, b) $\sin x$.

    6 points each.

11. Let $y = \text{cotan}(7x - 5) + 6$.

    a) Find all vertical asymptotes of the graph of y.

b) What is the period of y?

c) Describe *in words* how the graph of y can be obtained from the graph of $y = \text{cotan}(x)$. (A sketch of the graph is NOT required.)

Answer: a) $(n\pi + 5)/7$, b) period is $\pi/7$, c) horizontal (stretch/shrink) by a factor of 7, shift right by 5/7, vertical shift up 6.

a) 5 points, b) 3 c) 12 (3 points for each transformation and 3 for correct order) "

12. Find an algebraic expression (i.e., one that does not involve the trigonometric functions and their inverses) that is equivalent to the expression $\sin(\arctan(x))$.
answer: $x/(\sqrt{(}1 + x^2)$.
6 points for correct triangle; 6 for correct function.

13. Sam's bicycle tire has a radius of 13.5 inches. If he is riding at a speed of 20 feet/second, what is the angular speed of his tire, correct to 2 decimal places in revolutions/minute?
Answer: 14.15 rev. per minute
$1200/(2\pi * 13.5)$ 10 points for correct setup.

14. A tourist notices two window washers working on a side of a tall building in the Chicago Loop, one washer directly above the other. The angles of elevation from the tourist to the washers are 45° and 67°, respectively. The tourist is 120 feet from point at the base of the building that is directly beneath the washers. What is the distance between the two window washers?
Answer: $120(\tan 67° - \tan 40°) = 182.01 ft$.
9 pts for set up, 3 for evaluation

15. Express *all* of the 6^{th} roots of 64 in standard form $a + bi$, a, b real.
Answer: For k = 0 to 5, $2((\cos(2\pi + k)/6) + i\sin((2\pi + k)/6)$.
2 is worth 3. partial credit for getting some solutions (knowing they are placed around the circle.)