Math 503: Problems on Bouscaren paper

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Read the paper as soon as possible. There is nothing actually new until page 11. We will be discussing it by Friday at latest. These problems should be turned in next Monday. (Jan. 19)

1. What are the technical points referred to on the top of page 6. Be sure to ask if you have trouble with them.

2. Prove Fact 3.2 (the Tarski Vaught test).

3. Prove Fact 3.3 (2). (You can assume 3.3 1).)

4. Prove the assertion in the first paragraph at the top of page 12.

An ordered field $(F, +, \cdot, 0, 1, \leq)$ is called *Archimedean* if for any positive $a, b \in F$ there is an n with $a < n \cdot b$.

5. Does the notion of Archimedean ordered group make sense? How would you define it? What is different about an Archimedean field?

6. An element of an ordered field, k, is said to be *finite* if for some natural number n, $-n(1) \leq a \leq n(1)$. Suppose that for any formula $\phi(x)$, if $k \models (\exists x)\phi(x)$ then for some finite $a \in k$, $k \models \phi(a)$. Prove k is elementarily equivalent to an Archimedean field.