Math 503: 3rd problem set

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January 30, 1998

Question: if $A \prec B$ and $B \prec A$ must A and B be isomorphic?

Sources of counterexamples:

The theory of an equivalence relation with infinitely many infinite classes.

Same theory but with an additional binary relation which behaves as successor on each class.

The second is needed to get countable examples.

1. Find counterexamples both countable and uncountable.

Definition. M is weakly saturated if M realizes every n-type over the empty set (for every n).

Definition. M is ω-saturated if M realizes every n-type over any finite set contained in M (for every n).

Definition. M is κ-universal if $|N| < \kappa$ and N elementarily equivalent to M implies there is an elementary embedding of N into M.

2. What is the relationship among these three notions.

Definition. The theory T is small if for each n, $S_n(\emptyset)$ is countable.

3. Show that if complete countable theory T is small, there is a countable saturated model and there is an (elementarily) prime model.

4. What can you say about the converse to the preceding?