Math 503: 5th problem set

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due March 13.

For A and abelian group write A^κ for the κ-fold direct product of copies of A and $A^{(\kappa)}$ for the κ-fold direct sum of copies of A.

1. Show that if A is finite abelian group $A^{(\kappa)}$ is elementarily equivalent to A^κ and moreover, for infinite κ, the theory is independent of κ. (It might be easier to keep track of the main ideas by doing this for Z_p which is ok. There are more general results (cf. Feferman and Vaught) but this case can be done with Ehrenfeucht games.) I think the original result is Eklof and Fisher (Annals Math Log. 1972)

2. Show the theory of $Z_2^{(\omega)}$ is strongly minimal. Show the theory of $Z_4^{(\omega)}$ is $\aleph_1$-categorical but not strongly minimal.

3. Show that for any countable theory and any infinite κ, there is a model M of T of cardinality κ such that every definable subset of M has cardinality κ.

4. The quantifier elimination for the real field shows that the theory of real closed ordered fields is o-minimal. (Every definable subset is a finite union of points and intervals.) Use this (and other properties of the real field) to show that for any real closed field M with cardinality κ, every definable subset of M has cardinality κ. Is this theory $\aleph_1$-categorical?