Math 503: 7th problem set: Nonstandard Analysis

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due March 13.

1. Prove that a continuous function of a compact set is uniformly continuous.

2. Complete the proof (begun in class) of Tychonoff's theorem.

3. Suppose X is a hyperfinite set and f is an internal function from X to X. Prove f is 1-1 if and only if f is onto.

The following are from the Henson notes.

4. Let $a_1, \ldots a_n$ be internal in $V(^*S)$. Show $\{a_1, \ldots a_n\}$ and $\langle a_1, \ldots a_n\rangle$ are internal.

Definition: A nonstandard extension of $V(S)$ is an *enlargement* if for each set $a \in V(S)$ there is a hyperfinite set b such that b contains every standard element of $^*a$, i.e. $\{^*c : c \in a\} \subseteq b \subset^* a$.

5. The following are equivalent:

1. The nonstandard extension is an enlargement.
2. If $k \in \mathbf{N}$ and $\mathcal{F}$ is a collection of subsets of $V_k(S)$ with the finite intersection property, the $\bigcap\{^*a : a \in \mathcal{F}\}$ is nonempty.
3. If $k \in \mathbf{N}$ and $(L, \leq)$ is a partially ordered set in $V_k(S)$ which is directed upwards, then there exists a $b \in^* L$ such that for all $a \in L$, $^*a \leq^* b$.

Conclude every polysaturated nonstandard extension of $V(S)$ is an enlargement.