Math 503: 8th problem set: Strength of Nonstandard Analysis

John Baldwin

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due April 20.

1. Prove the following assertion is true in ${}^*PA$.

$$(\forall a \in N)(\forall b)[b \leq a \rightarrow N(b)].$$

2. The $\aleph_1$-saturation schema is:

$$(\forall X)[[card(X) < \aleph_1$$

$$\&(\forall Y)(\exists a)(\forall b \in Y)(Y \text{is finite } \& Y \subset X) \rightarrow \phi(a,b))] \rightarrow (\exists a)(\forall b \in X)\phi(a,b)].$$

Note that this is property of 'local saturation' since only one formula ϕ is involved in each schema.

Prove that if (A, S_1) is a model of ${}^*PA^{(2)}$ and S_1 is the full power set of A, then A is $\aleph_1$-saturated in the normal sense of the word. (Hint: (with thanks to Keisler). Note that A has cofinality ω_1. So each set of countably many formulas which is finitely satisfiable is finitely satisfiable below some c. The definable subsets of the set of predecessors of c can be (uniformly) coded by divisiblity. Now use 'local saturation' for the divisibility predicate.