Finite Model Theory, Spring 1997

Assignment 1. Due approximately Feb. 10

1. Choose a language L so you can write a sentence with the following property. (L will contain a unary predicate symbol P, constant symbols 0, 1, and a linear order $<$ of $\neg P$.) There is an L sentence ψ such that if A is a finite model of ψ, $\neg P(A)$ is a sequence of 0's and 1's which expresses the cardinality of $P(A)$ in binary. (There will be other symbols in L; it is helpful to have a linear order of P.)

2, Fix a finite relational language L. For any class K of L-structures $\mathbf{L}(K) = \{B : \mathrm{every substructure} A \mathrm{of} B \in K\}$; $\mathbf{S}(K) = \{B : B \subseteq A \mathrm{for some} A \in K\}$. Show that if $K = \mathbf{L}(K) = \mathbf{S}(K)$ then K is axiomatized by a set of universal first order sentences.

3. Show using Ehrenfeucht games that any two infinite discrete linear orders without endpoints are isomorphic.

4. Exercise 1.3.11 of Ebbinghaus and Flum (page 24).

5. Exercise 1.3.12 of Ebbinghaus and Flum (page 24).

6. Exercise 6.2.5 of Ebbinghaus and Flum (page 133). Show that in a) of this problem 2(m+1) can be improved to 2 log m if structures are codes as in Vaananen instead of Ebbinghaus and Flum.