STEPHEN WOLFRAM. ***A New Kind of Science.*** Wolfram Media, Inc., Champaign, IL, 2002, xiv + 1197 pp.

Wolfram proclaims the investigation of A New Kind of Science. The study of computer generated mathematical phenomena is surely a kind of science which arose in the last 20 years. While investigations in e.g., complex dynamical systems have pursued the 'old' scientific methods of using (computer-generated) examples to motivate precise mathematical definitions and prove theorems, Wolfram takes a more descriptive approach. On the basis of examining literally billions of configurations he proclaims certain uniformities. There is no attempt in the book to either define or prove in the usual mathematical sense of the term. Indeed, identifying the perspective from which he writes poses a major difficulty for understanding. 'Randomness' is a central concept of the book. But it is only on pages 512–516 that one learns that Wolfram means by random a rather loose notion of pseudorandom: meets common statistical tests. Rather than guessing his meanings I use words as they are commonly used by logicians. Accordingly, I will describe as 'conjectures' certain statements which Wolfram characterizes as 'proven' or just asserts are true. References to some of the best of the many reviews of this book appear in *A mathematician looks at Wolfram's new kind of science* by L. Gray in ***Notices of the American Mathematical Society***, vol. 50 (2003), pp. 200–211; I confine myself to issues raised that have a specifically logical tinge as they concern the foundations of computation. In particular, I am avoiding any issues in the philosophy of science.

In general a cellular automaton acts on a k-dimensional space (Z^k) with each cell labeled by one element of a finite vocabulary V, by simultaneously updating each cell in accord with a rule that uses only the contents of the cells within radius r. More particularly, the basic objects of most of Wolfram's analysis are *pictures* generated as follows. A two state one-dimensional radius 1 cellular automaton acts on an (in practice finite) string of black and white cells. At each stage each cell is updated by a rule which depends on the previous value of itself and its immediate neighbors. Each iteration represents one row in a two dimensional picture. Wolfram developed a useful scheme of numbering these automata. For example, Rule 110 preserves the color of a cell unless the cell is white and one or both of its neighbors is black.

We consider six 'conjectures', which are discussed in the book.

CONJECTURE 1. *There is a two state, one dimensional, radius one cellular automaton* (*rule* 110) *which is capable of universal computation.*

The truth of this assertion depends on the Church's thesis like problem of defining 'universal computation'. The ability to simulate a universal Turing machine is a weaker demand than the ability to simulate a universal cellular automaton (which has infinite input). Matthew Cook has provided a mathematical argument that rule 110 can simulate all Turing machines. Even this is an extraordinary result. It says that one can simultaneously minimize all three parameters of cellular automata complexity (vocabulary, dimension, radius) and still have a universal machine for computing recursive functions. Wolfram provides a sketch of the argument, which is due to Cook. (Read Gray *op. cit.* for an account of the fascinating controversy concerning ownership of this result.) In contrast to earlier constructions of Turing machines designed to be universal, Cook takes a given simple machine and argues that it is universal.

But there are further logical questions about the meaning of universal, which are raised, for example by B. Durand and Zs. Rókas *The game of life: universality revisited*, in Delorme and Mazoyer, editors, ***Cellular Automata: A parallel model***, vol. 460 of Mathematics and Its applications, Kluwer Academic, 1998. The essential problem is whether the encoding and decoding might do all the work leaving a 'universal machine' which is a fraud. This particular encoding and decoding is carefully described by Cook, though not in Wolfram's book. The preprocessor is given a tape with a code for a Turing machine program and an input. Successively encoding this information by a tag system and a cyclic tag system, we arrive at the input for an infinite tape. It consists of a finite segment and cyclic information at each end. The finite segment and the cycle depend on *both* the program and the specific input. This type of coding appears to be accepted in the 'small universal Turing machine' community. But three issues arise. First, give technical arguments that in a precise sense the coding and decoding do not add too much information. (Martin Davis did this for Turing machine computability in *A note on universal Turing machines*, in C. E. Shannon and J. McCarthy, editors, ***Automata Studies***, vol. 34 of Annals of Mathematical Studies, pp. 167–176, Princeton University Press, 1956.) Second this simulation requires an exponential slow down (cf. B. Durand and Zs. Rókas *op. cit.*). Finally, even if the coding and decoding are considered technically benign, the supposed simplicity of rule 110 is impaired by the recognition that it uses extensive problem-dependent pre- and post-processing.

CONJECTURE 2. *Rule* 30 *produces pseudorandom sequences.*

Since this is the tool for 'random generation' in Mathematica, this must be true in the 'meets statistical tests' sense of Wolfram. But it is unclear whether any formal argument for computational indistinguishability in the sense of e.g., O. Goldreich, (***Modern Cryptography, Probabilistic Proofs, and Pseudorandomness***, Springer-Verlag, Berlin-Heidelberg-New York, 1999) has been given.

Wolfram's consistent generalization to 'all cellular automata' of properties discussed only for low dimensional automata amounts to:

CONJECTURE 3. *The behavior of* 1 *and* 2 *dimensional cellular automata of small radius and vocabulary is typical of the behavior of all cellular automata.*

I almost write assumption rather than conjecture for this item; the only hint at an argument for it is that such complexity is found in these cases that surely they must represent all possibilities.

CONJECTURE 4. *All cellular automata can be classified into* 4 *categories.*

This conjecture has two parts.

CONJECTURE 4a. *There are four types of pictures.*

The types are described vaguely and perhaps inconsistently at various points in the book. In short the pattern dies out, is periodic, random, or 'exhibits complicated structure'. The inconsistency arises as in the first chapter 'self-similar' patterns are identified. These appear highly determined but seem to be later classed as 'random'. (Does this inconsistency disappear with the Wolfram meaning of 'random'?) In Wolfram's earlier ***Cellular Automata and Complexity***, Addison Wesley, 1994, several more precise attempts at a definition are presented; this precision has disappeared in this book, which is intended for a popular audience. Gray *op. cit.* suggests at least 6 or 7 different types of pictures.

CONJECTURE 4b. *There are four types of cellular automata.*

A picture depends on a cellular automata (rule) and an input. Culik and Yu *Undecidability of ca classification schemes*, ***Complex Systems***, vol. 2 (1988), pp. 177, translated the classification of pictures to a classification of automata by a 'worst case' analysis; Baldwin and Shelah *On the classification of cellular automata*, ***Theoretical Computer Science***, vol. 230 (2000), pp. 117–129, showed that a classification of the form, "rule x is class y if with probability z it has class y behavior" is not possible. Wolfram takes a decidedly biological view of the classification. 'And in a sense the situation was similar to what is seen, say, with the classification of materials into solids, liquids, or gases, or of living organisms into plants and animals. At first a classification is made on the basis of general appearance. But later, when more detailed properties become known, these properties turn out to be correlated with the classes that have already been identified. Often it is possible to use such detailed properties to make more precise definitions of the original classes. And typically all reasonable definitions will then assign any particular system to the same class.' (p. 235). Unfortunately, there is little evidence in the book for the last sentence.

CONJECTURE 5. *The above classification of automata determines their computational power.*

Class 4 automata are asserted to be 'capable of universal computation'. There are serious problems with this conjecture. As noted above one has to make both the classification and the notion of universal computation precise. Easy operations on cellular automata preserve universality (in some sense) but change the classification. Wolfram makes no distinction between the ability to *simulate* an arbitrary Turing machine (cellular automata) and the ability to *compute* every recursive function. Wolfram freely uses the term universal computation when (I believe) universal simulation is meant. On page 717 of the book under review Wolfram, we find:

CONJECTURE 6. *The principle of computational equivalence*: *'almost all processes that are not obviously simple can be viewed as computations of equivalent sophistication'*

Process here is an open-ended term including physical processes. But it seems quite strong enough for logical purposes if we restrict ourselves to cellular automata. In particular this should imply a precise conjecture that most (all) class 4 automata are 'computationally equivalent'. More stunning to a logician is that as noted by H. Cohn in *Review of Wolfram: A New Kind of Science*, ***MAA online***, 2002, this leaves no room for the intermediate degrees. Perhaps, Wolfram's intent is that the machines which compute intermediate degrees are 'unnatural'. Or perhaps this is an effect of the fundamental divergence between the meaning of 'compute' for, say, Turing and for Wolfram. A Turing machine computes a function. For Wolfram, a cellular automaton just computes. He raises a fundamental question of comparing two computing processes, not some classical function that they compute.

In summary, Wolfram provides a number of fascinating insights and speculations into the behavior of cellular automata. Formalizing and determining the truth of these suggestions could be a rewarding task for logicians. In particular, (i) give a satisfying formalization of the notion of a *universal* cellular automata, (ii) find, if possible, formal connections between the dynamic behavior of computation by a machine (roughly Wolfram's classification) and the complexity of computations it can do. Cellular automata may become a paradigm for parallel computing complexity as Turing machines are for serial computing complexity. But to explore that avenue, look not at Wolfram but at complexity theory work as in the volume edited by Delorme and Mazoyer, ***Cellular Automata: A parallel model***, vol. 460 of Mathematics and Its Applications, Kluwer Academic, 1998.

JOHN BALDWIN

Department of Mathematics, Statistics and Computer Science, University of Illinois at Chicago. jbaldwin@uic.edu.

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