

The Math Forum PEMDAS and FOIL

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We describe the Mathematics Forum of the University of Illinois at Chicago and try to draw some lessons for the professional development of secondary school mathematics teachers. The first section is a quick sketch of the workings of the forum. The second tries to place it in context and analyze some of the opportunities and difficulties. We conclude with a more detailed description and some problems from one session.

1 Description

The Algebra Symposium/Math Forum began in the Spring of 2005. It began as course co-taught by 5 research mathematicians on the teaching of algebra. For reasons that I will discuss in the Section 2, it evolved into a series of monthly meetings that took on a wider focus, hence the math forum. The principal aspects of the math forum as it evolved over the last 4 years were:

1. Monthly meetings
2. Mostly high school teachers; some middle school
3. 2-5 faculty at each session
4. 20-40 teacher participants

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5. stipends, food, parking, conversation
6. various topics related to high school mathematics

The program was aimed at secondary mathematics teachers; there were two or three middle school and two or three special education teachers who attended fairly regularly. The sessions were led by mathematicians but were structured with many small group activities. At various times 8 members of the research faculty at UIC presented; but two organizers (Baldwin, Masley) were the only constant presence. Eventually, we started each fall with a featured speaker (Danny Martin, Allison, Mark Driscoll). For the last two years two of our clinical faculty who supervise student teachers have been frequent attendees. And there has been sporadic attendance from professional developers at the Chicago Public Schools (CPS). The program was advertised on CPS websites, the UIC Mathematics Education website, through a mailing list that grew to over 100 and by word-of-mouth. Until the last year, teachers received \$25 per hour stipends from an NCLB grant.

Here are some sample topics.

1. Calculators and Student Understanding
2. Mnemonics (PEMDAS, FOIL, etc.) and Their Impact on Student Success
3. Real Life Situations, Modeling, and the Fundamental Theorem of Algebra
4. Mathematics Success and Failure Among African American Youth
5. Interpreting Political Statistics -the GINI index
6. Expectations and Assessment
7. Teachers' Moves and Decisions and Students' Understanding

2 Context and Analysis

The Mathematics Forum is an attempt to reach individual teachers across a large district on a monthly basis. The topics are broadly drawn from mathematics and are not tied to any specific curriculum nor did they have

close support in individual high schools. The motivations for attendance are Illinois CPDU credits (widely available elsewhere), stipends, food and conversation, and the program content.

UIC has an active program of Continuing Education courses for middle school courses. These provide teachers with courses that prepare them for the Illinois Middle School Endorsement. Such an endorsement is necessary to teach mathematics courses as opposed to teaching mathematics in a self-contained classroom. Thus, this is a valuable credential which requires substantial mathematics beyond the mathematics necessary for a K-9 certificate. Thus there are clear career reasons for taking further coursework in mathematics.

The situation for high school teachers is quite different. Teachers who obtain endorsement as undergraduate mathematics (or in some cases mathematics education) majors or (programs like our MST-secondary) make up something less than 1/2 of the mathematics teachers in the Chicago High Schools. About 30% hold standard teaching credentials with a mathematics endorsement -this can be obtained with 20 semester hours of low level mathematics. A number of those without the mathematics endorsement are teaching special education courses in mathematics.

The Math Forum tries to provide a place for teachers to meet and discuss mathematics. It targeted a broad range of mathematical backgrounds by focusing on notions that were relevant to the classroom. And it tries to push for a deeper understanding of fundamental mathematical ideas that arise in high school classes. These discussions bring out somewhat unexpected gaps in knowledge: Must the altitude of a triangle lie within the triangle? Is there a difference between the slope of a line and the slope of an equation? Many activities were stimulated by earlier teacher feedback. For example, when we asked how Pythagorean Theorem lessons went, they told us they had to start with exponents because there were going to be students who had 3 squared being 6. In some sense the PEMDAS-FOIL items came up as examples of teachers needing to unteach some rules learned earlier.

With a well-organized systematic program and stipends, we were able to maintain an attendance of around 20 for several years. A less systematic program (4 sessions instead of 6) and no stipends was less successful this year. The kinds of questions that arise in the sessions demonstrate how essential such mathematical support is. But it needs to be integrated with school programs, with a summer component, and perhaps with course work.

3 Pemdask/Foil session

The following agenda for the Feb. 2, 2006 session on Pemdask and Foil illustrates the general style of meetings. However, the careful attempt to measure the effect of the evening was unique. Pemdask and Foil are acronyms that are taught to help students with the order of operations and with multiplication of binomials. But there are numerous hazards in using these mnemonics. We discussed some of these and general issues of the role of mnemonics. For example, many, but by no means all, of the teachers realized the difficulty that PEMDAS (Parentheses, exponents, multiplication, division, addition, subtraction) may lead students to perform all additions before subtractions.

Agenda: Pemdask/Foil

- 4:30 - 4:45 Introduction, Pre-Survey
- 4:45 - 5:30 PEMDAS session
- 5:30 - 5:45 Mentor Program and announcements
- 5:45 - 6:00 Debrief and summary 6:00 - 6:30 Dinner/Discussion
- 6:30 - 7:05 FOIL session
- 7:05 - 7:20 Debrief and summary
- 7:20 - 7:30 Post-Survey, Forms

At the beginning of the PEMDAS and of the FOIL session teachers were given a survey. The second just replaced 'Foil' by 'PEMDAS' in the sample we now give.

1. Are you familiar with the mnemonic Foil, 'first, outer, inner last'?
2. If the answer is yes?
 - (a) Have your students learned this mnemonic in elementary school?
 - (b) Do you teach it?
 - (c) Does the mnemonic always give the correct answer?
 - (d) Do you think this mnemonic is
essential counterproductive
1 2 3 4 5

The teachers were asked to work in small groups on the following problem set. Then the group came back together to report their results and to consider the problems on the second page following.

PEMDAS: Order of Operations

*Parentheses, Exponents, Multiplication
Division, Addition, Subtraction*

I. A student asks how he is supposed to divide before he adds for the problem: $\frac{2+3}{4+1}$

How do you proceed?

What should "twenty divided by the sum of five and fifteen" be? Why?

II. Is $12/4 * 3 = 12/4(3)$? See if your calculator agrees. What should a student get for $12/4x$ when $x = 3$? How do you justify your answer?

III. What is $-x^2$ when $x = -3$? What is $(-x)^2$ when $x = -3$? What is $(^-x)^2$ when $x = -3$? How do you explain?

IV. Why do some teach students Order of Operations?

What problems have you encountered in algebra based on student understanding or misunderstanding of order of operations?

What answer do you expect for $6 \log(100) * 10^2$?

What if the asterisk is not there?

V. What's the difference between a convention and a law? Is PEMDAS (Please Excuse My Dear Aunt Sally) a set of laws? Is it complete?

Masley led a general discussion about those problems and the following examples. A selection of calculators were available; various of them gave different answers to some of the problems listed below.

PEMDAS: Examples

$$5 + 3 \cdot 4^2$$

$$(5 + 3) \cdot 4^2$$

$$3 \cdot 2 + 5 - 2^2$$

$$3 \div 2 + 5 - 2^2$$

$$3 \cdot y + 5 - x^2$$

$$3 \div y + 5 - 2^2$$

$$3 \div (2 + 5) - 2^2$$

The session on FOIL (First, outer, inner, last) was stimulated by a college senior in the secondary education program being unable to multiply a binomial times a trinomial. She didn't know how to 'foil it'. We tried to place the multiplication of binomials into the context of later high school mathematics. We emphasized that a procedure can be so well adapted for the task at hand that it inhibits the students work on related problems. Here are the problem sets for small-group discussion of FOIL.

FOIL

I. A student is asked to multiply $(x + 2)(x^2 + 3x + 4)$ and writes:

$$x^3 + 4x + 2x^2 + 8.$$

He explains his answer by saying, 'I foiled it.'
How do you proceed?

Ia) What are several explanations for the method of multiplying polynomials. Is there more than one method?

II. Why do we teach students to multiply binomials?

III. Why do we teach students to factor trinomials?

IV. How should one explain the procedure for computing

$$(x + y)^3?$$

V. How do we prevent the error: $\sqrt[4]{x^4 + y^4} = x + y?$

4 Effect of PEMDAS/FOIL session

We repeated the introductory survey after the discussions. Thus the pre-post survey gives an unusual opportunity to see if there was at least temporary effect of a session that had very specific objectives.

Almost all said students had learned PEMDAS in elementary school. About half said FOIL had been learned.

The presurvey revealed that 17 of the attendees taught PEMDAS, while 11 did not. But 9 taught FOIL while 12 did not (the foil survey was administered after dinner).

Participants ranked PEMDAS and foil from 1 (essential) to 5 (counter-productive) before and after the session.

The average score moved on PEMDAS from 2.85 to 3.17 and on FOIL from 3.38 to 3.81.

Both of these shifts were statistically significant at the 1% level.

Three of six no longer think Pemdask always give correct answer.

Each meeting of the Math Forum concluded with filling out evaluations that are part of the State of Illinois continuing professional development program.

Our overall average on all sessions: 4.7

Pemdask/foil session 4.6.

Strikingly, the score of 4.3 on connections with state teaching standards was the lowest entry. Is this a flaw in the standards? Or do people not like to change their minds?

5 Two More Problems

We conclude with two more exciting problems. This is the actual announcement sent to the teachers.

Geometry in Tunisia February 8, 2007

John Baldwin will lead a discussion of several problems from a daily math/physics column in 'La Presse de Tunisie'.

The height of a lighthouse

A lighthouse is visible from a distance of 32 km. At what height above sea-level is the lamp?

What are three interpretations of this problem. Why can you solve all three (with slightly different answers depending on your interpretation)?

We will discuss this and two other problems from the paper. If anyone has any background on why problems of this sort appear in the daily press, we'd like to know.

John went to Tunisia during his sabbatical in Spain. The Sahara is a truly fascinating place and you might enjoy the travelogue at:
<http://www2.math.uic.edu/~jbaldwin/tunisia.pdf>

The following problem entranced the mathematicians. Solutions and partial solutions involving standard geometry, trigonometry, and geometric sketch pad circulated among 5 mathematicians. I got still another solution from one of the participants in the Tucson meeting.

If in a triangle, the angle bisector is also the bisector of the angle between the altitude and the median issuing from the same vertex, what can one say about this triangle.

The teachers liked both of these problems but felt that even the first was far too hard for their students.