A Numerical Scheme for the Quantum Boltzmann Equation Efficient in the Fluid Regime

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Abstract:

Numerically solving the Boltzmann kinetic equations with the small Knudsen number is challenging due to the stiff nonlinear collision term. A class of asymptotic preserving schemes was introduced in \cite{FJ10} to handle this kind of problems. The idea is to penalize the stiff collision term by a BGK type operator. This method, however, encounters its own difficulty when applied to the quantum Boltzmann equation. To define the quantum Maxwellian (Bose-Einstein or Fermi-Dirac distribution) at each time step and every mesh point, one has to invert a nonlinear equation that connects the macroscopic quantity fugacity with density and internal energy. Setting a good initial guess for the iterative method is troublesome in most cases because of the complexity of the quantum functions (Bose-Einstein or Fermi Dirac function).

In this talk, we propose to penalize the quantum collision term by a `classical' BGK operator instead of the quantum one. This is based on the observation that the classical Maxwellian, with the temperature replaced by the internal energy, has the same first five moments as the quantum Maxwellian. The scheme so designed avoids the aforementioned difficulty, and one can show that the density distribution is still driven toward the quantum equilibrium. Numerical results are present to illustrate the efficiency of the new scheme in both the hydrodynamic and kinetic regimes. We also develop a spectral method for the quantum collision operator.