A WEIGHTED DISPERSIVE ESTIMATE FOR SCHRÖDINGER OPERATORS IN DIMENSION TWO

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Consider the two-dimensional linear Schrödinger equation with potential,

$$iu_t(x,t) + Hu(x,t) = 0, \qquad u(x,0) = f(x).$$

Here $H = -\Delta + V$, where V is a real valued potential on $\mathbb{R}^2$ satisfying $|V(x)| \lesssim \langle x\rangle^{-3-}$. When $V = 0$, it is well-known that the solution operator satisfies the mapping estimate $\|e^{-it\Delta} f\|_\infty \lesssim |t|^{-1}\|f\|_1$. With sufficient assumptions on the potential V and the spectrum of H, one can prove a corresponding bound for the perturbed equation, $\|e^{itH}P_{ac}f\|_\infty \lesssim |t|^{-1}\|f\|_1$.

In dimensions one and two it is possible to obtain faster decaying estimates at the cost of weights. We prove that if zero is a regular point of the spectrum of $H = -\Delta + V$, then

$$\|w^{-1}e^{itH}P_{ac}f\|_{L^\infty(\mathbb{R}^2)} \lesssim \frac{1}{|t|\log^2(|t|)}\|wf\|_{L^1(\mathbb{R}^2)}, \qquad |t| > 2,$$

with $w(x) = \log^2(2 + |x|)$. This decay rate was obtained by Murata in the setting of weighted L^2 spaces with polynomially growing weights. This is joint work with Burak Erdoğan.