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A Logarithmic Diffusion Equation as the Limit of Porous Medium Equations

Abstract The solution to $u_t = \Delta \ln u$ can be viewed as a formal limit of the solutions to the porous medium equations $u_t = \Delta \frac{u^m}{m}$. Recently some authors made such a limit rigorous by prescribing initial or/and boundary data. However our approach is entirely local (joint work with E. DiBenedetto and U. Gianazza). Under the assumption that

$$\frac{u_m^m - 1}{m} \in L^p_{loc}, u_m \in L^r_{loc}$$

for some $p > N + 2$ and $r > \frac{1}{2}N$ where u_m is the solution to $u_t = \Delta \frac{u^m}{m}$ and N is the space dimension, we establish a $C^{\alpha, \frac{1}{2}\alpha}_{loc}$ limit process by finding the uniform upper bound and lower bound of solutions to the porous medium equations. The uniform lower bound is realized by a Harnack-type inequality.