69TH MIDWEST PARTIAL DIFFERENTIAL EQUATIONS SEMINAR ABSTRACTS

Christof Sparber and **Peter Markowich**

Title. On Wigner and Bohmian Measures

Abstract. We consider the time-evolution of quantum particles described by the linear Schrödinger equation in a semi-classical scaling. We will report on recent results on so-called Bohmian measures and their classical limit. Bohmian measures describe the time-evolution of the quantum mechanical position and velocity densities and are given by the push-forward under the Bohmian flow on phase space. The latter can be seen as a perturbation of the classical Hamiltonian flow. Connections to the, by now, classical theory of Wigner measures are also discussed.

Deniz Bilman

Title. Long time asymptotics for perturbations of the Toda lattice

Abstract. We consider one dimensional doubly-infinite Fermi-Pasta-Ulam (FPU) lattices obtained via perturbations of the completely integrable Toda interaction potential. We investigate, via numerical methods, the long time asymptotics of these nearly integrable systems. In particular, our aim is to relate the asymptotic resolution into solitary waves in the perturbed systems to the soliton resolution in the Toda system.

Thierry Colin

Estimating the growth of metastasis starting from medical imaging

The aim of the talk is to present a model of brain tumor involving several population of cells. We will show haw we can recover the main features observed in MRI, especially concerning the invasive characters of glioblastoma. This is a joint work with H. Fathallah (UAB, Birmingham), O. Saut (Bordeaux, France), J.-B. Lagaert (Grenoble, France).

Sergey Denissov

On the instability in two-dimensional fluid

Abstract: For the two-dimensional Euler equation, we prove lower bounds for the growth of Sobolev norms. We also explain the merging of two centrally symmetric vortex patches and prove the double-exponential estimates for infinite time.

Speaker: Burak Erdoğan

Title: Smoothing for the KdV equation and the Zakharov system on the torus.

Abstract: In this talk we will consider the periodic KdV equation and the periodic Zakharov system in one dimension. We prove that the difference of the nonlinear and the linear evolutions is in a smoother space than the initial data. The method is based on normal form calculations and $X^{s,b}$ space estimates. We will also discuss applications such as almost everywhere convergence to initial data, growth bounds for higher order Sobolev norms, and the existence and smoothness of global attractors. This is a joint work with Nikos Tzirakis.

Alex Himonas: *"The initial value problem for the CH and DP equations"*.

Abstract. We shall consider the Cauchy problem for the Camassa-Holm (CH) and Degasperis-Procesi (DP) equations and discuss their well-posedness properties in Sobolev spaces H^s. For $s > 3/2$ these equations are well-posed in the sense of Hadamard. When $s < 3/2$, both CH and DP are ill-posed in Sobolev spaces H^s. This will be the main focus of our presentation. The talk is based on work in

collaboration with Carlos Kenig, Gerard Misiołek, Curtis Holliman and Katelyn Grayshan.

Speaker: Peter Hinow

Title: Size-structured populations with distributed states at birth

Abstract: Age-structured models based on first order hyperbolic partial differential equations have been employed successfully in population dynamics for a long time and are considerably well understood. In contrast to such models where every individual is born at the same age 0, size-structured models allow to take into account different, distributed birth sizes. "Size" here can be a quite general concept, for example mass, energy content or pathogen load in a disease model. This introduces a birth operator that takes values in an infinite-dimensional Banach space and complicates greatly the mathematical analysis. In this survey, we will describe some examples of models that we recently investigated in a series of joint papers with Jozsef Farkas (University of Stirling, United Kingdom). The emphasis will be on questions such as asymptotic growth for linear models and the existence and stability of steady states for nonlinear models.

Speaker: Peter Miller

Title: The Benjamin-Ono Equation in the Zero-Dispersion Limit

Abstract: The Benjamin-Ono equation is a model for several physical phenomena, including gravity-driven internal waves in certain density-stratified fluids. It has the features of being a nonlocal equation (the dispersion term involves the Hilbert transform of the disturbance profile) and also of having a Lax pair and an associated inverse- scattering algorithm for the solution of the Cauchy initial-value problem. We will review known phenomena associated with this equation in the limit when the dispersive effects are nominally small, and compare with the better-known Korteweg-de Vries equation. Then we will present a new result (joint with Zhengjie Xu) establishing the zero- dispersion limit of the solution of the Benjamin-Ono Cauchy problem for certain initial data, in the topology of weak convergence. Our methodology is a novel analogue of the Lax Levermore method in which the equilibrium measure is given more-or-less explicitly rather than via the solution of

a variational problem. The proof relies on aspects of the method of moments from probability theory.

Speaker: Dorina Mitrea

Title: Boundary Value Problems: Higher Order Regularity Data in Nonsmooth Settings

Abstract: Methods based on pseudodifferential calculus have proved vastly successful in dealing with boundary value problems in smooth domains, but they have the drawback of crucially relying on smoothness. At the other end of the spectrum, with the advent of the modern Calderón-Zygmund theory of singular integral operators, the theory of elliptic boundary value problems in Lipschitz domains has presently reached a remarkable degree of sophistication.

One basic issue left unresolved at the present time is reconciling these existing theories (dealing, respectively, with very smooth and very irregular geometries) by developing an all-encompassing theory which contains the aforementioned ones as limiting end-points. This talk will partially address this issue by focusing on the solvability via boundary integral methods of the Dirichlet problem for elliptic systems under optimal geometric measure theoretic assumptions and with boundary data belonging to higher order Sobolev spaces.

Katharine Ott: *The mixed problem in Lipschitz domains.*

Abstract. In this talk I will discuss the mixed problem, or Zaremba's problem, in a bounded Lipschitz domain. Consider $\Omega \subset \mathbb{R}^n$, $n \geq 2$, a bounded Lipschitz domain with boundary $\partial\Omega$ decomposed as $\partial\Omega = D \cup N$, with D and N disjoint and D an open subset of $\partial\Omega$. We specify Dirichlet boundary data on D and Neumann boundary data on the remainder of $\partial\Omega$. The problem reads

$$\mathcal{L}u = 0 \text{ in } \Omega, \quad u = f_D \text{ on } D, \quad \frac{\partial u}{\partial \nu} = f_N \text{ on } N. \tag{0.1}$$

Above, $\mathcal{L}$ is a second order elliptic elliptic operator with constant coefficients and ν is the outward unit normal vector defined a.e. on the boundary. We seek conditions on the domain, the boundary, and the

data which guarantee that the gradient of the solution of (0.1) lies in $L^p(\partial\Omega)$ for $1 \leq p < \infty$. I will discuss recent results of this nature when the underlying operator $\mathcal{L}$ is the Laplacian or the Lamé system of elastostatics. This is joint work with Russell Brown and Justin Taylor.

Speaker: Katelyn Grayshan

Title: Peakon solutions and continuity properties of the Novikov equation

Abstract. We shall consider both the periodic and non-periodic Cauchy problem for the Novikov equation and discuss continuity results for the data-to-solution map in Sobolev spaces. In particular, we show that the data-to-solution map is not (globally) uniformly continuous in Sobolev spaces with exponent less than 3/2. To accomplish this, we construct sequences of peakon solutions whose distance initially goes to zero but later becomes large.

A WEIGHTED DISPERSIVE ESTIMATE FOR SCHRÖDINGER OPERATORS IN DIMENSION TWO

WILLIAM R. GREEN

Consider the two-dimensional linear Schrödinger equation with potential,

$$iu_t(x,t) + Hu(x,t) = 0, \qquad u(x,0) = f(x).$$

Here $H = -\Delta + V$, where V is a real valued potential on $\mathbb{R}^2$ satisfying $|V(x)| \lesssim \langle x\rangle^{-3-}$. When $V = 0$, it is well-known that the solution operator satisfies the mapping estimate $\|e^{-it\Delta}f\|_\infty \lesssim |t|^{-1}\|f\|_1$. With sufficient assumptions on the potential V and the spectrum of H, one can prove a corresponding bound for the perturbed equation, $\|e^{itH}P_{ac}f\|_\infty \lesssim |t|^{-1}\|f\|_1$.

In dimensions one and two it is possible to obtain faster decaying estimates at the cost of weights. We prove that if zero is a regular point of the spectrum of $H = -\Delta + V$, then

$$\|w^{-1}e^{itH}P_{ac}f\|_{L^\infty(\mathbb{R}^2)} \lesssim \frac{1}{|t|\log^2(|t|)}\|wf\|_{L^1(\mathbb{R}^2)}, \qquad |t| > 2,$$

with $w(x) = \log^2(2 + |x|)$. This decay rate was obtained by Murata in the setting of weighted L^2 spaces with polynomially growing weights. This is joint work with Burak Erdoğan.

Title: On uniqueness of heat flow of harmonic maps and hydrodynamic flow of nematic liquid crystals

Authers: Tao Huang (Department of Mathematics, University of Kentucky)
with Changyou Wang (Department of Mathematics, University of Kentucky)

Abstract: We establish the uniqueness of the heat flow of harmonic maps that have sufficiently small renormalized energies. As corollaries, we obtain (i) the uniqueness of heat flow of harmonic maps whose gradients belong to $L_t^p L_x^q$ for $q > n$ and (p, q) satisfying Serrin's condition, and (ii) the uniqueness for hydrodynamic flow (u, d) of nematic liquid crystals, with $u, \nabla d$ satisfying Serrin's condition.

Naian Liao

A Logarithmic Diffusion Equation as the Limit of Porous Medium Equations

Abstract The solution to $u_t = \Delta \ln u$ can be viewed as a formal limit of the solutions to the porous medium equations $u_t = \Delta \frac{u^m}{m}$. Recently some authors made such a limit rigorous by prescribing initial or/and boundary data. However our approach is entirely local (joint work with E. DiBenedetto and U. Gianazza). Under the assumption that

$$\frac{u_m^m - 1}{m} \in L^p_{loc}, u_m \in L^r_{loc}$$

for some $p > N + 2$ and $r > \frac{1}{2}N$ where u_m is the solution to $u_t = \Delta \frac{u^m}{m}$ and N is the space dimension, we establish a $C^{\alpha, \frac{1}{2}\alpha}_{loc}$ limit process by finding the uniform upper bound and lower bound of solutions to the porous medium equations. The uniform lower bound is realized by a Harnack-type inequality.

A numerical and analytical studies of solitary-wave solution for the Extended Benjamin-Bona-Mahony equation

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ABSTRACT

The Regularized Long-Wave equation (RLW-equation), also known as the BBM-equation $u_t + u_x + (u^2)_x - u_{xxt} = 0$, was first studied as a model for small-amplitude long waves that propagate on the free surface of a perfect fluid [1]. As an alternative to the Korteweg-de Vries equation, it features a balance between nonlinear $(u^2)_x$ and frequency dispersive terms $-u_{xxt}$ that allow existence of traveling waves that are smooth and symmetric about their maximum. Such waves decay rapidly to zero on their outskirts and, because of their nature to travel alone, are known as solitary waves.

We are interested here in solitary-wave solutions of the equation

$$u_t + \alpha u_x + \beta_p(u^p)_x + \beta_q(u^q)_x - \gamma u_{xxt} = 0,$$

which we named the EGRLW-equation (Extended Generalized Regularized Long Wave-equation). We are investigating change of stability of such solutions for different powers p and q, as well as various values of the coefficients α, β_p, β_q and γ. Presented here are numerical simulations of evolution of solitary waves, the behavior of solitary waves under perturbation as well as interaction of two solitary waves and resolution of the initial disturbances into solitary waves.

References

[1] T.B. Benjamin, J.L. Bona and J.J. Mahony Model equations for long waves in nonlinear dispersive systems, *Philos. Trans. Royal Soc. London Ser. A*, **272** (2007.07.17)

Spectral Properties of the Reflection Operator in Two Dimensions

Eric Stachura

Abstract

We study spectral properties of the reflection operator (a singular integral operator arising naturally in connection with the radiosity equation which models the energy transfer between different parts of a surface by radiation) acting on L^p spaces, $p \in (1, \infty)$, on infinite angles in two dimensions. More specifically we establish an explicit characterization of the spectrum and spectral radius estimates for the reflection operator acting on L^p spaces on an infinite angle in two dimensions. This type of analysis is relevant to the solvability of the radiosity equation with L^p data since when the spectral radius is $<$ 1, the solution can be explicitly expressed as a convergent Neumann series.

Mixed Boundary Value Problems for Quasilinear Elliptic Equations

Chunquan Tang

Abstract: We study the boundary value problems for general quasilinear elliptic equations with mixed Dirichlet and oblique boundary conditions. We obtain a gradient estimate for solutions under various structure conditions on the operators and domains. A special case is the following capillary problem

$$\begin{cases} \operatorname{div}\left(\frac{Du}{\sqrt{1+|Du|^2}}\right) + B(x,u,Du) &= 0, \qquad \text{in } \Omega \\ u &= \phi(x), \quad \text{on } \partial_1\Omega \\ \frac{Du\cdot\gamma(x)}{\sqrt{1+|Du|^2}} &= \cos\beta, \quad \text{on } \partial_2\Omega \end{cases}$$

where $\gamma(x)$ is the unit inner normal on $\partial_2\Omega$. Suppose θ_0 is the largest angle formed by $\partial_1\Omega$ and $\partial_2\Omega$. We show that, among other conditions, if $\theta_0 < \frac{\pi}{2} - \left|\frac{\pi}{2} - \beta\right|$, a global gradient bound exists.

BOUNDS FOR THE EIGENVALUES OF THE FRACTIONAL LAPLACIAN AND STOKES OPERATOR

SELMA YILDIRIM YOLCU

1. Abstract

This talk focuses on some eigenvalue inequalities for fractional Laplacian operators $(-\Delta)^{\alpha/2}$, $\alpha \in (0, 2]$ (a prototype of non-local operators) and Stokes operators by using some of the tools used to prove analogues results for the eigenvalues of the Laplacian. (joint work with T.Yolcu)

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