Mixed Boundary Value Problems for Quasilinear Elliptic Equations

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Abstract: We study the boundary value problems for general quasilinear elliptic equations with mixed Dirichlet and oblique boundary conditions. We obtain a gradient estimate for solutions under various structure conditions on the operators and domains. A special case is the following capillary problem

$$\begin{cases} \operatorname{div}\left(\frac{Du}{\sqrt{1+|Du|^2}}\right) + B(x,u,Du) = 0, & \text{in } \Omega \\ u = \phi(x), & \text{on } \partial_1\Omega \\ \frac{Du\cdot\gamma(x)}{\sqrt{1+|Du|^2}} = \cos\beta, & \text{on } \partial_2\Omega \end{cases}$$

where $\gamma(x)$ is the unit inner normal on $\partial_2\Omega$. Suppose θ_0 is the largest angle formed by $\partial_1\Omega$ and $\partial_2\Omega$. We show that, among other conditions, if $\theta_0 < \frac{\pi}{2} - \left|\frac{\pi}{2} - \beta\right|$, a global gradient bound exists.