Math 215 -Fall 2010 - Assignment #6

1. Read Chapters 10, 12,13,14.
Read for the ideas, but particularly look at Chapter 14 in the light of our work on Cantor and infinite sets. Chapter 10 you should read now for the parts that you recognize. There is some new material such as the principle of inclusion-exclusion that we will do later. Chapter 11 as it is written is not what I want to use, and so we will talk about properties of finite sets in a different but similar way in class. Chapter 12 is about fundamental combinatorics of counting, and we will do some of this in class. Look at the chapter for reference. Chapter 13 should be familiar from our class work. Chapter 14 should be mostly familiar from our class work.

2. Read the notes from the book by John Conway that are on the website. These notes are about ordering infinities and we will discuss this in class.

3. Consider the following list of sequences of 0's and 1's.

1. 11111111111111111111111111111111111111...
2. 01111111111111111111111111111111111111...
3. 00111111111111111111111111111111111111...
4. 00011111111111111111111111111111111111...
5. 00001111111111111111111111111111111111...
6. 00000111111111111111111111111111111111...

...

In general, the n-th row of this list consists in (n-1) 0's followed by infinitely many 1's. If you apply the Cantor diagonal method to this list you get the diagonal is

111111111111111111111111....

and its flip is

000000000000000000000000....

As we know the flip of the diagonal is a new sequence that is not on the original list. Now add this new sequence to the top of the list and get a new flipped diagonal. Keep on doing this.

...

5'. 0011000000000000000000000000000000000000...
4'. 1110000000000000000000000000000000000000...
3'. 01000000000000000000000000000000000000 ...
2'. 1000000000000000000000000000000000000000...

1'. 00000000000000000000000000000000000000...
1. 11111111111111111111111111111111111111...
2. 01111111111111111111111111111111111111...
3. 00111111111111111111111111111111111111...
4. 00011111111111111111111111111111111111...
5. 00001111111111111111111111111111111111...
6. 00000111111111111111111111111111111111...
...

As we discussed in class, it appears that the pattern of the new sequences is that the first column alternates 0 and 1. The second column alternates 00 and 11. The third column alternates 000 and 111. And so on. Prove by induction that this is true.

Take another example of an infinite sequence of sequences, and use it, via Cantor's diagonal process, as a generator for a new list of sequences. See if you can find a pattern in your new list.

4. Let Seq{0,1} denote the set of all infinite sequences of 0's and 1's. Let Zeq{0,1} denote the set of all infinite sequences of 0's and 1's such that each sequence in Zeq{0,1} is eventually, I.E. after a finite number of terms, all 0's. Prove that Zeq{0,1} is a countable set. Use the method by which you have listed the elements of Zeq{0,1} and apply Cantor's diagonal process to that list. You should produce a sequence that is not on the list, AND your new sequence should not be in Zeq{0,1}. Check that it is not in Zeq{0,1}.

5. Use the LK notes on discrete calculus on our website for this problem. First extend the following table:

$n = n^{(1)}$

$n^2 = n(n-1) + n = n^{(2)} + n^{(1)}$

$n^3 = ?$

$n^4 = ?$

In other words, express n^3 and n^4 in terms of $n^{(1)}$, $n^{(2)}$, $n^{(3)}$ and $n^{(4)}$.

Now use your extended table and the discrete calculus to find a formula for $H(n) = 1^4 + 2^4 + 3^4 + ... + n^4$. To do this first note that $H(n+1) - H(n) = (n+1)^4$, and apply the discrete calculus to this difference to solve the problem.

6. Page 87, problems 7.5, 7.6.
Page 113, problem 9.6.
Page 116, Problems 8, 9, 11, 12, 14.
Page 184, Problem 13. Do this problem as stated in the text and also do it by the method we explain in class for finding greatest common divisors.