Math 215 -Fall 2010 - Assignment #7
Due Wednesday, November 24, 2010.

1. Read Chapters 15,16,17 (we have done all this in class).
Read Chapters 19,20,21 (this will be discussed in the week of November 15 - 19).

2. page 225. Problems 2,3,5,6

3. For the primes p = 2,3,5,7,11,13
make a chart for each prime showing the possible rmainders on division by p of the square of a natural number. For example, for p=5, we have

$0^2 = 0$, remainder = 0
$1^2 = 1$, remainder = 1
$2^2 = 4$, remainder = 4
$3^2 = 9 = 5 + 4$, remainder = 4
$4^2 = 16 = 15 + 1$, remainder = 1
$5^2 = 25 = 25 + 0$, remainder = 0

The pattern repeats after this, and so we have only the remainders 0,1,4. The numbers 2 and 3 do not appear as remainders on division of a square by 5. The problem asks you to determine this information for each prime in the set above (for division of squares by that prime number.)

4. This problem uses continued fractions which we define as follows.
$[a] = a$
$[a,b] = a + 1/b$
$[a,b,c] = a + 1/(b + 1/c)$
and recursively by
$[a_1, a_2, a_3, ..., a_n] = a_1 + 1/[a_2, a_3, ..., a_n]$.

(a) Prove by induction that

$$[a_1, a_2, a_3, ..., a_n, x]= (Rx + S)/(Px + Q)$$

where R,S,P and Q are functions only of $a_1, a_2, ..., a_n$ (and not invlolving x).

(b) Show that $x = [a_1, a_2, a_3, ..., a_n, x]$ is always a quadratic equation. Solve explicitly $x = [1,2,3,4,x]$.

(c) Examine evidence (i.e. try some examples) for the

Theorem. If $a_1, a_2, \ldots, a_n$ are positive integers and $[a_1, a_2, a_3, \ldots, a_n] = P/Q$ where P and Q are positive integers with no common factor, then $[a_n, a_{n-1}, a_{n-2}, \ldots, a_2, a_1] = P/S$ where P and S are relatively prime positive integers and $QS \equiv (-1)^{n+1} \pmod{P}$.

For example [1,3] = 1 + 1/3 = 4/3
[3,1] = 3 + 1/1 = 4/1
and 1 x 3 = 3 == -1 (mod 4).
In your examples try some longer continued fractions like [1,2,3,4,5].