Proof in Mathematics

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It may help to distinguish the two types of proof that mathematicians use.

Type One: Derivations that stepwise proceed from a set of axioms to a conclusion. Here every single step is justified by explicit rules, and the entire proof could be checked by a computer.

Type Two: Informal arguments that are understood to be expandable to derivations of Type One.

In the type two proof (and most proofs in the literature are of type two) one has to rely on the properties of the domain of enquiry, the rigor and impeccability of the language used to form the proof. Such proofs are like recipes. You read the proof and follow the instructions therein. Upon doing so, you may become completely convinced of the logicality of the argument. A lot of work is left to the reader. There is no way that a computer can check such a proof unless the computer has a PhD level of understanding.

There is a tradeoff between rigorous understanding, allowing communication, and derivation by rule, allowing detailed automatic checking. We try to wend a path that goes between and can be translated into the completely stepwise form.

In either case, all mathematical proofs are relative to some set of assumptions that are not proved (the axioms). Thus mathematics never really 'proves' anything and since these axioms are formal relationships of undefined terms, we also are not proving anything about any thing. This led Bertrand Russell to say "Mathematics is the subject where we do not know what we are talking about nor whether what we are saying is true."