Hour Exam 2 — Math 310 (Applied Linear Algebra) — 12 PM section — April 11, 2008

Show all of your work! An unjustified answer is not correct.
Put all of your work and answers on the blank paper handed out.

1) [20 pts] Consider the following matrix

$$A = \begin{bmatrix} 1 & 2 & 1 & 3 \\ 2 & 3 & 1 & 5 \\ 3 & 4 & 1 & 7 \end{bmatrix}$$

as a linear transformation from R^4 to R^3.
(a) Let $Col(A)$ denote the range of A. That is, $Col(A) = \{Ax\}$ where x runs over all vectors in R^4. Determine a basis for $Col(A)$ and find the dimension of $Col(A)$.
(b) Let $S = Col(A)^{\perp}$ be the subspace of R^3 orthogonal to the range of A. Find a basis for S. What is the dimension of S?
(c) Find a basis for the row space of A.

2) [20 pts] Let L be the linear transformation from R^2 to R^3 given by the following equation: $L(x, y)^T = (x + y, x - y, y - x)^T$.
(a) Let M denote the matrix of L with respect to the standard bases for R^2 and R^3. Determine the matrix M.
(b) Let $E = [u_1, u_2] = [(1, 1)^T, (1, -1)^T]$ be a new basis for R^2 and let $F = [b_1, b_2, b_3] = [(1, 1, 1)^T, (1, 1, 0)^T, (1, 0, 0)^T]$ be a new basis for R^3. Find the matrix A for L with respect to these bases.

3) [20 pts] Let V be the space of real-valued differentiable functions of the variable x spanned by $\{e^x, xe^x\}$. Let $D : V \longrightarrow V$ be the linear transformation d/dx (derivative with respect to x).
(a) Show that $\{e^x, xe^x\}$ are linearly independent in V. Note that V is a subspace of the space of all differentiable functions of a real variable x. Addition in W is addition of functions. Scalar multiplication is the multiplication of a function by that scalar.
(b) By part (a), $\{e^x, xe^x\}$ is a basis for V. Find the matrix of D with respect to this basis.

4) [20 pts] Let $L : R^2 \longrightarrow R^2$ be the linear transformation whose matrix in the standard basis is

$$A = \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}.$$

(a) $v_1 = (2, -1)^T$ and $v_2 = (1, -1)^T$. Verify that $E = [v_1, v_2]$ is a basis for R^2.

(b) Find the matrix $B = [L]_E^E$. This is the matrix for L in the basis E. Check your answer.

5) [20 pts] (a) Let Π be the plane in R^3 defined by the equation $x+y+z = 0$. Find a general formula for the distance of a point $P = (x, y, z)^T$ to the plane. Use your formula to find the distance from $(1, 1, 1)^T$ to the plane.

(b) Let u, v and w be three non-zero vectors in R^3 such that each pair $\{u, v\}$, $\{u, w\}$ and $\{v, w\}$ is orthogonal. Show that $[u, v, w]$ is a basis for R^3. Verify this using only the properties given for these vectors. Do not use specific numerical examples.