The Brasilian Knot Trick

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1 Introduction

In this paper we introduce a significant variant on the famous Chefalo knot trick [1] that we dub the Brasilian knot trick. It happened this way: The first author of this paper (a Brasilian of Italian descent), whom we shall refer to as MEB was being shown the Chefalo knot by second author, whom we shall refer to as LK. All this happened long ago and far away in the year 1996 at an International Conference on Mathematical Physics and Knot Tricks held at Cargese on the island of Corsica in the midst of a startlingly blue sea. But we digress. LK having demonstrated the disappearing nature of the Chefalo knot, said to MEB "Now you do it!" MEB took up the rope and tied a knot that we saw at once was not the Chefalo knot. But even though it was not, it was in fact not a knot and this new knot that was not a knot became the Brasilian knot- the subject and object of our paper.

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2 Brasil Knots

We begin with Figure 1, a depiction of the original Brasilian knot as tied by MEB on the lunch table in Corsica, under the noonday sun by the deep blue sea. It surprised us a bit to see this knot appear. MEB thought it quite natural at the time. LK was perturbed, but determined to push on. And so he said, "Well maybe your Brasilian version can still be threaded to make an unknot that will amaze us!".

And so they looked. And it did make an unknot by a little further threading just as shown in Figure 2.

But now, says LK, we have to add the ring, and find a threading that will be unknotted *and* release the ring. Must we? says MEB. We must, says he. See Figure 3.

And eventually they did. It took some time. And the stars showed over the deep blue sea. The ringed release knot is shown in Figure 4. Try it and you will see.

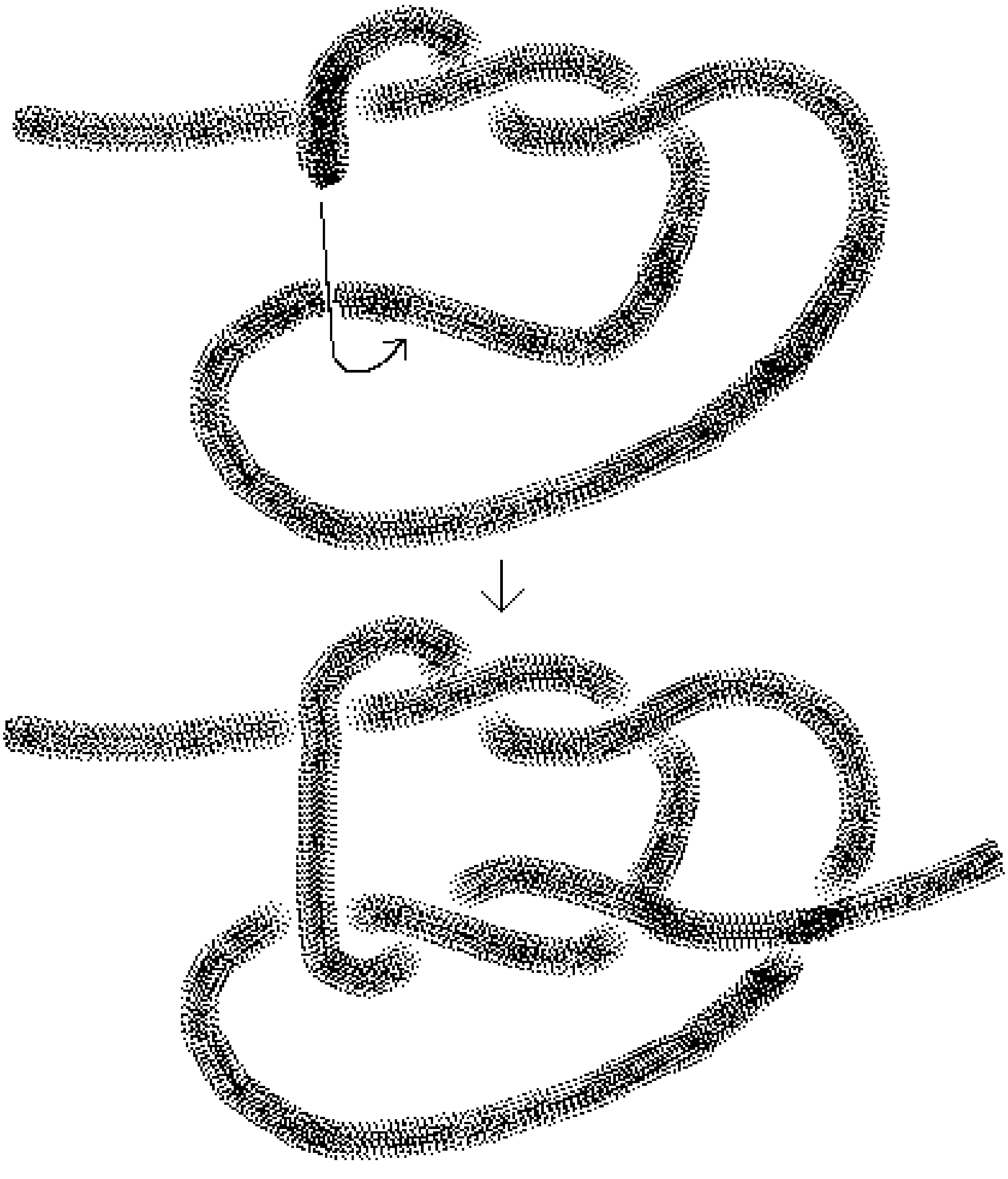

Figure 1: Brasilian Knot

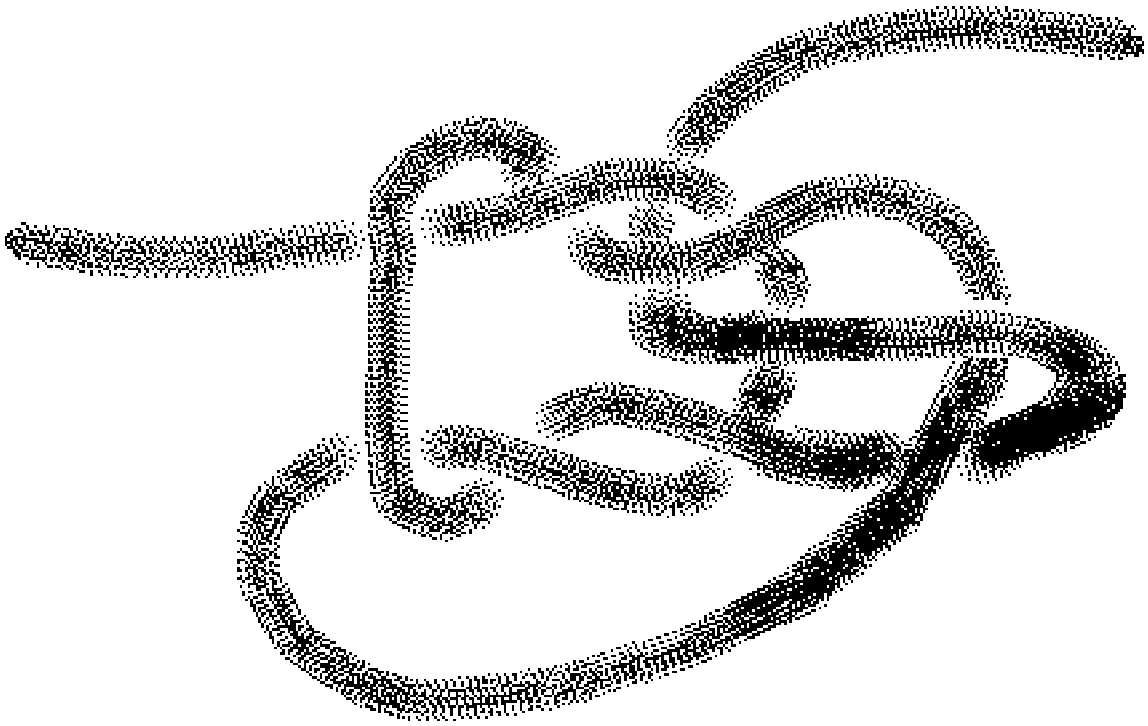

Figure 2: Brasilian UnKnot

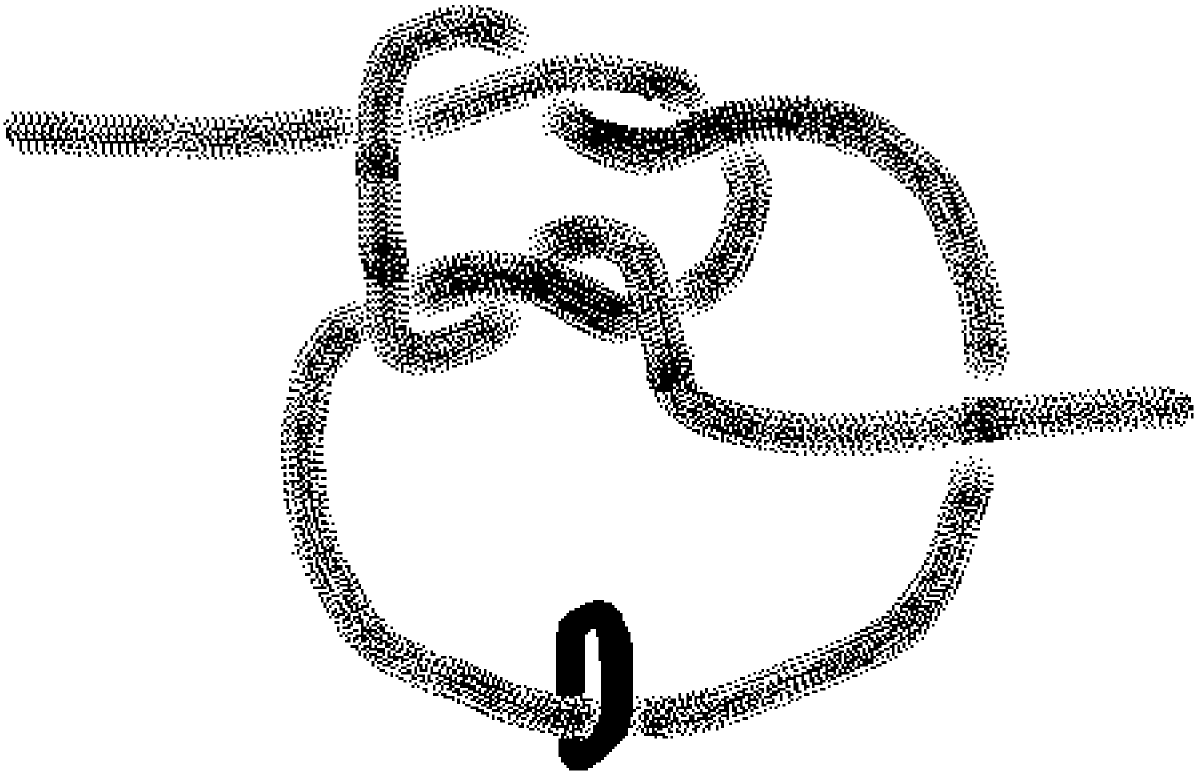

Figure 3: Brasil With a Ring

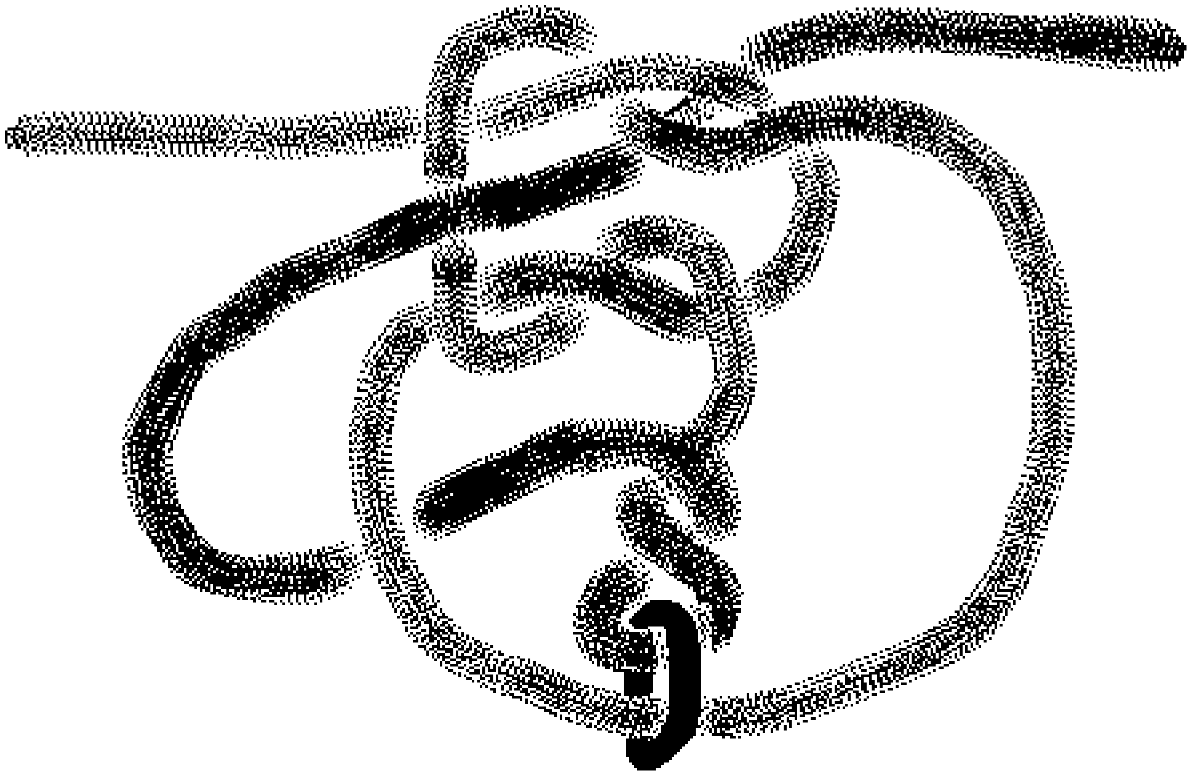

Figure 4: The Brasilian Unlinking Ring

3 A Few More Twists

There are infinitely many Brasilian knots and unlinking rings! In the construction of the Brasilian knot we made two windings, each a full turn, on the upper and lower portions of a bend in the rope. The second winding can be done any number of times and it can also be done in reverse. This means that there is a Brasilian knot and knot trick for each integer number N where N denotes the number of times one performs the second winding. The original Brasilian knot and unknot are the case of $N = 1$.

The reader who looks closely at our pictures of the various cases will note that a winding labelled N full twists consists in $2N+1$ half twists. Normally, $2N$ half twists make N full twists, but in the handling of the rope one begins the winding on one side of a strand, and ends it on the other side (imagining the rope laid nearly flat on a table). This movement from side to side creates the extra half twist in the winding.

Figure 5 shows the case that we call $N = -1$, a Brasilian analogue of the Granny knot. In this Figure we show both the knot and the unknot for the case of $N = -1$.In Figure 6 the ring release is shown for this case. It is interesting to note that the Granny knot version of the original Chefalo trick does not in any obvious way lead to an unknot or to a ring release. See the Appendix for a discussion of this point. In Figure 7 we show the case of the ring release for $N = 2$ and indicate the general case of arbitrary N. Figure 8 shows the Brasilian Unknot in the case $N = 2$.In Figure 9 we show the setup for the ring release with $N = 2$. We trust that these examples and a little ropework will convince the interested reader that there is a Brasilian knot trick for every integer (including zero)! A combination of $N = 1$, $N = -1$ and $N = 2$ makes a spectacular demonstration.

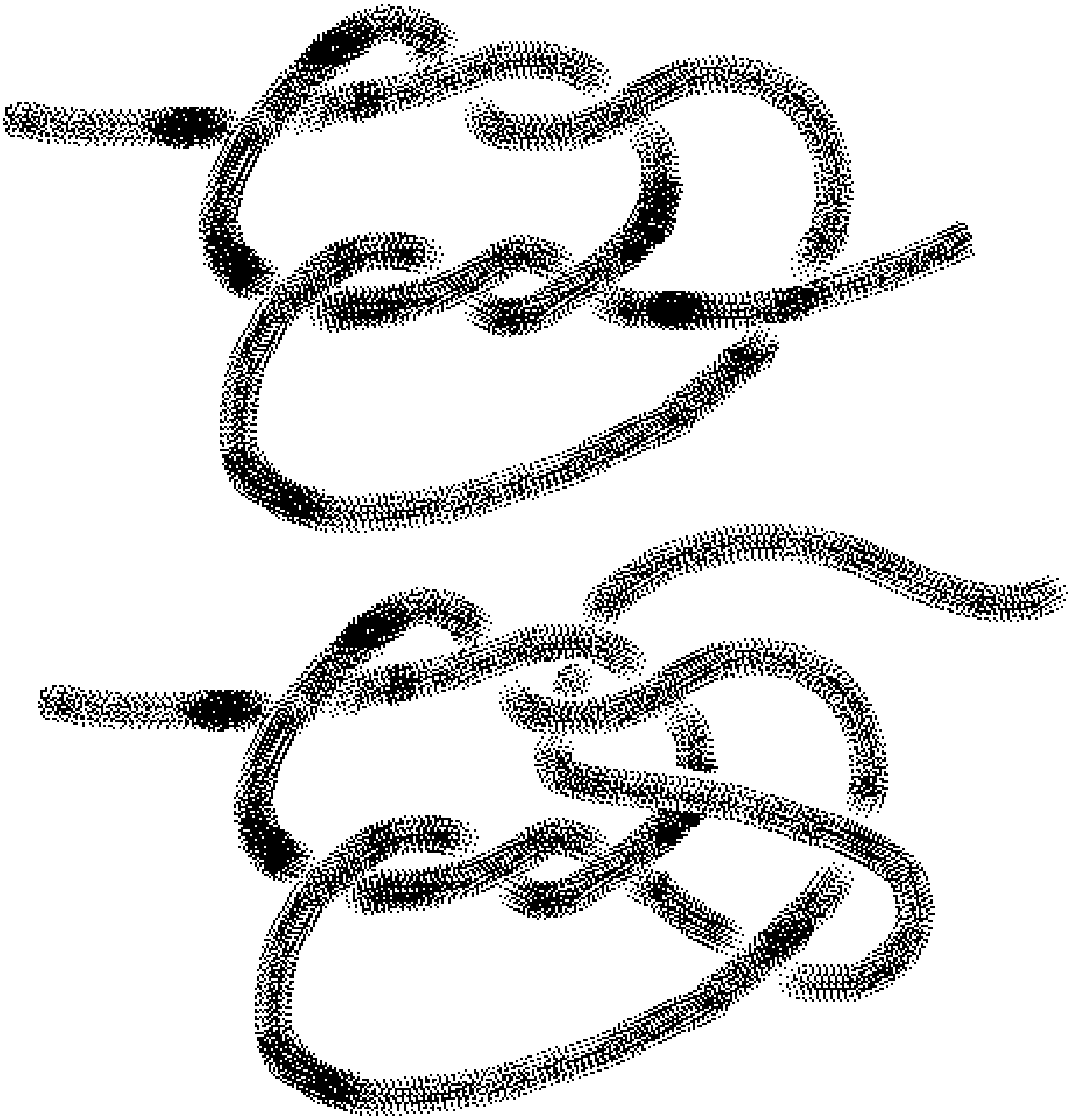

Figure 5: $N = -1$: The Brasilian Granny Knot

Figure 6: Ring Release for $N = -1$

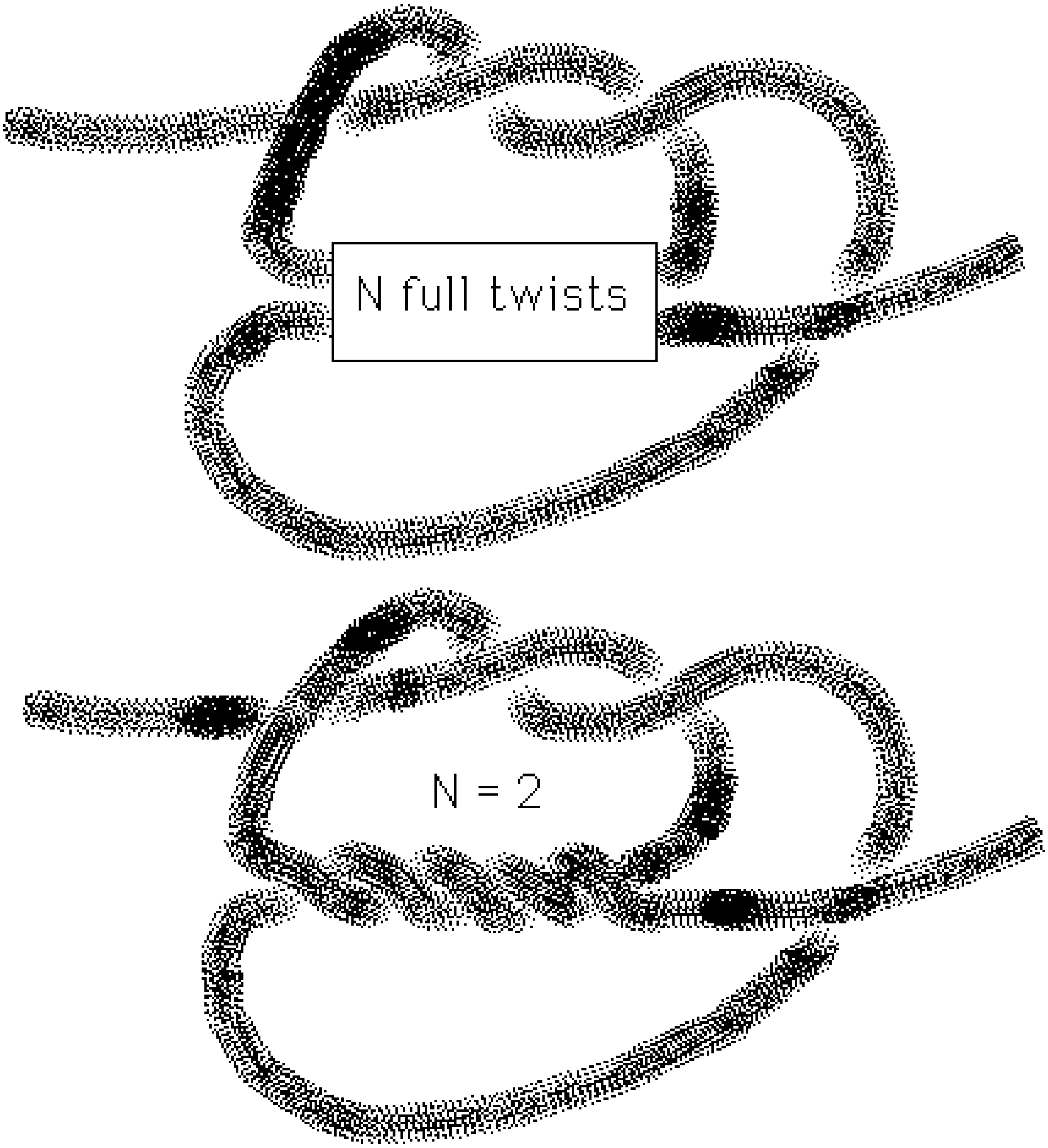

Figure 7: General N and $N = 2$

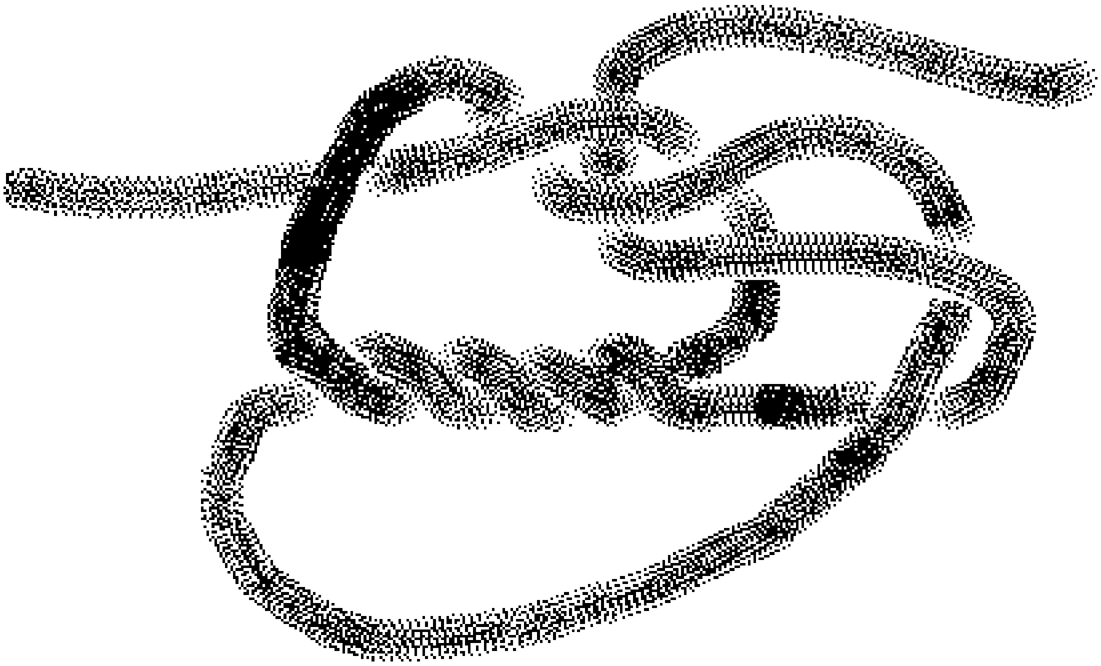

Figure 8: The $N = 2$ Brasilian Unknot

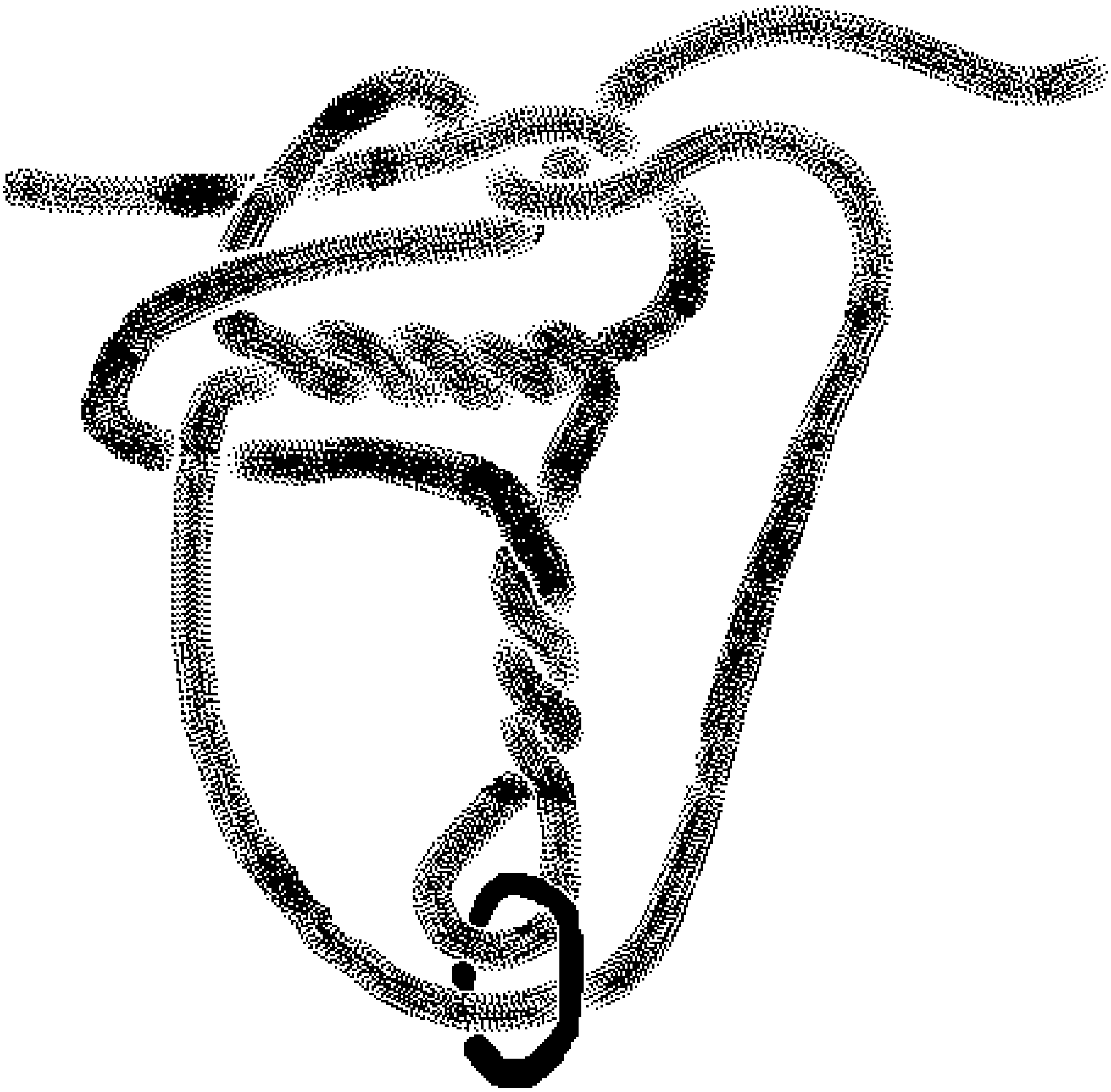

Figure 9: The Ring Release for $N = 2$

4 A Note on Performance

The knots and unknots herein described can be performed by tying them on a length of rope whose ends are then held fast by an assistant to the magician or by the magician herself. An unknot is demonstrated to be unknotted by pulling the ends of the rope until the "knot" in the middle disappears. A ring is demonstrated to be unlinked with the rope by pulling the ends, while an assistant helps the mass of rope in the middle disentangle by a little gentle encouragement, until the ring drops off and the rope is seen to be unknotted. Another method in all cases is to tie the ends of the rope together securely. The unknot or ring-release unknot can then be manipulated at will by the magician or spectator until it reveals its secret!

5 Appendix on the Chefalo Knot Trick

The original Chefalo knot trick is performed by starting with a version of the square knot as shown in Figure 10. The corresponding unknot is also shown in Figure 10. In Figure 11 we show the Granny knot as a starting position. There is no direct analogue of the unknot and ring release known to us that starts from the traditional Granny. The Brasilian Granny as shown in Figure 5 is our case $N = -1$ and works fine. The Brasilian Granny does the trick.

References

[1] C. W. Ashley, *The Ashley Book of Knots*, Doubleday, New York, London, Sydney, Auckland (1944).

[2] Cecile DeWitt-Morette, Pierre Cartier and Antoine Folacci (editors), *Functional Integration - Basics and Applications*, NATO ASI Series - Proceedings of the Nato Advanced Study Institute held at the Institut d'Etudes Scientifiques de Cargese from September 1 to 14, 1996. Plenum Press, New York and London (1997).

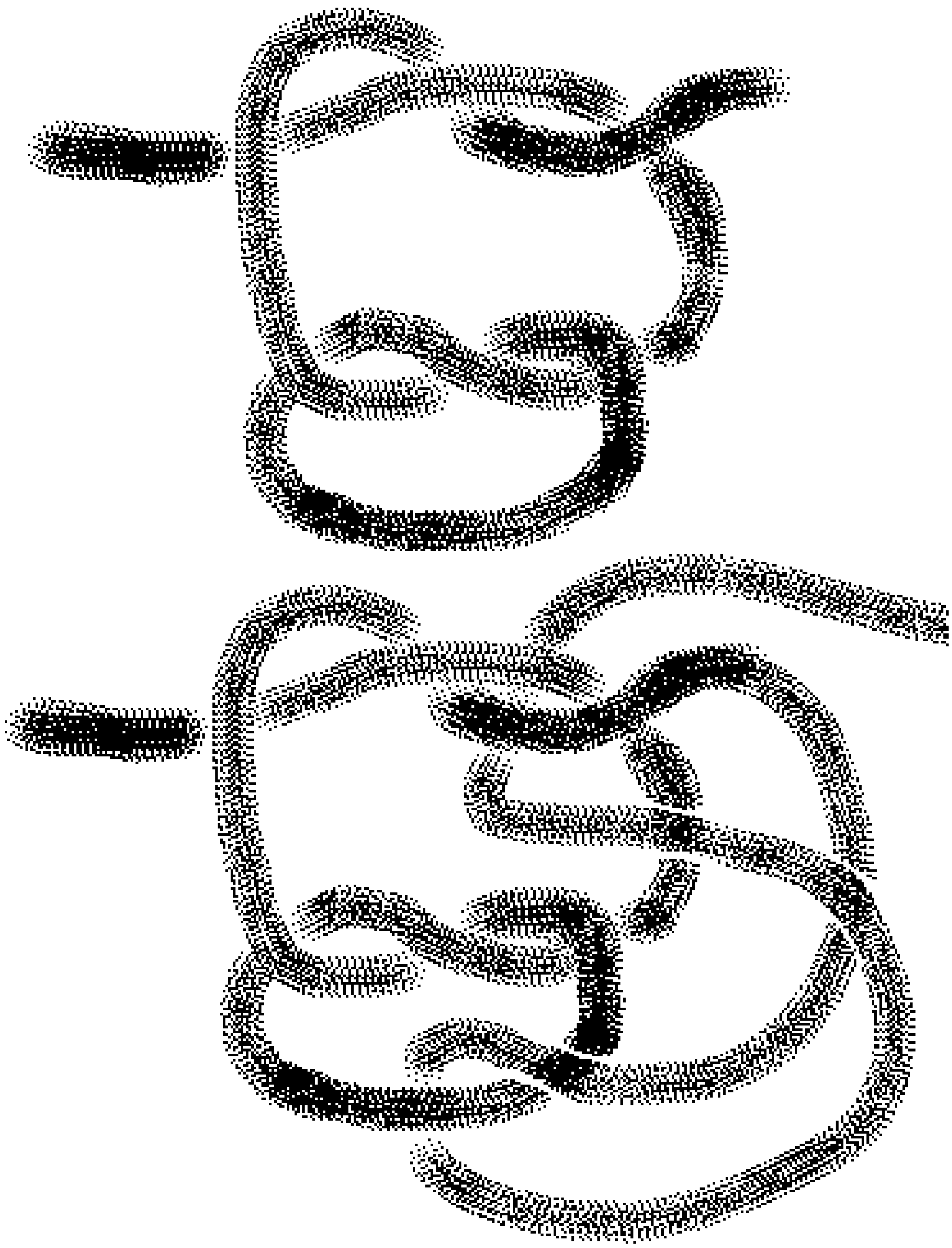

Figure 10: The Chefalo Unknot

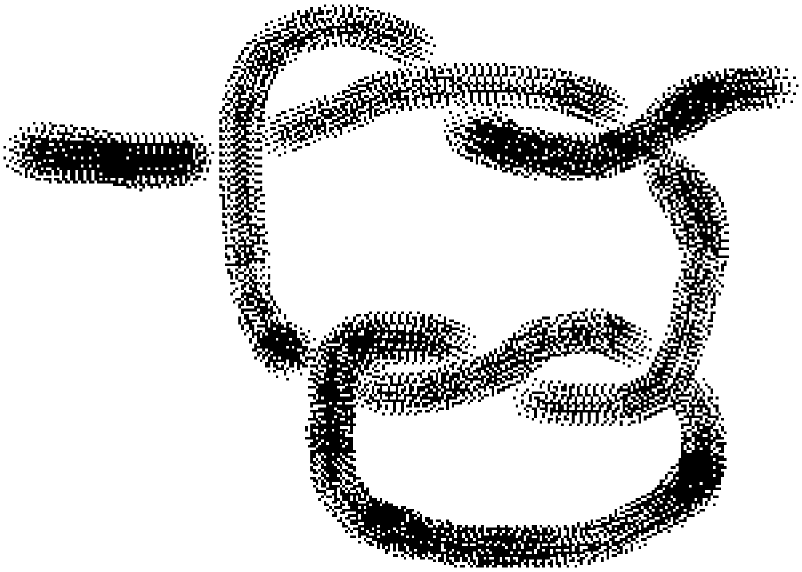

Figure 11: The Traditional Granny