LINKS WITH TRIVIAL JONES POLYNOMIAL

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ABSTRACT

Examples of 2- and 3-component links are given that cannot be distinguished from the corresponding unlink by means of the Jones polynomial.

Keywords: Link, Jones Polynomial

A fundamental problem in the theory of the Jones polynomial, unsolved as of this writing, is the question as to whether there exists a non-trivial knot with Jones polynomial equal to that of the unknot. In this note we present examples of 2- and 3-component links whose Jones polynomial is equal to that of the corresponding unlink. These examples, which appeared during the course of a routine computer enumeration, are all satellites of the Hopf link.

We recall that the Jones polynomial of the n-component unlink is u^{n-1}, where $u = -t^{-1/2} - t^{1/2}$, the Jones polynomial of the 2-component unlink.

Our first two examples are 2-component links with 15 and 17 crossings, each with Jones polynomial equal to u.
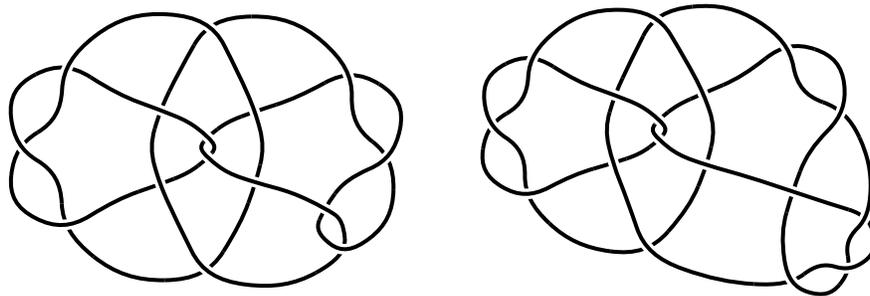
We remark that these links may be distinguished from one another (and from the unlink) by the knot types of their components.

Each of the next two examples, with 14 and 17 crossings respectively, has Jones polynomial equal to either $t^6 u$ or $t^{-6} u$, depending on choice of string orientations. Thus the polynomials are not equal to the unlink polynomial, but are close enough to warrant attention.
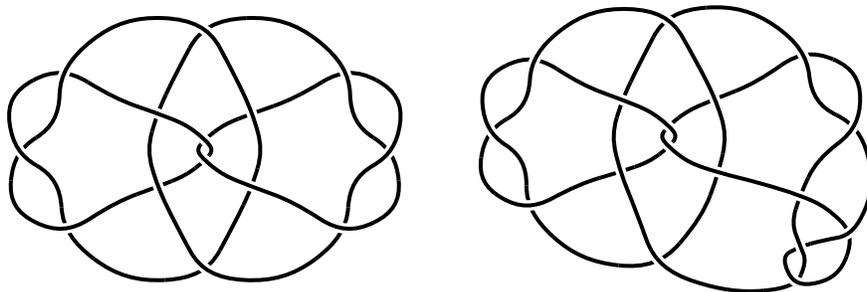
Our final example is a 16-crossing 3-component link whose Jones polynomial is u^2, on condition that orientations of the components are chosen so that the writhe of the diagram is -2.
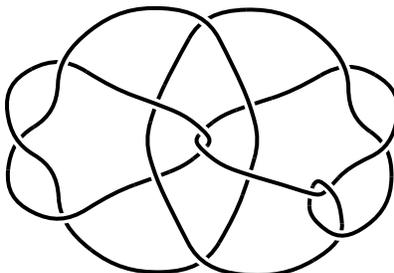
Exhaustive examination of the tables of links shows that these presentations of the five links all have minimal crossing-number.

Subsequent investigations, in collaboration with Shalom Eliahou and Louis H. Kauffman [EKT], have revealed the existence of several infinite families of 2- and 3-component links with trivial Jones polynomial.

It is a pleasure to acknowledge Kenneth Stephenson's Circlepack program [S], which played an essential role in generating the pictures in this article.

References

[EKT]   Shalom Eliahou, Louis H. Kauffman and Morwen B. Thistlethwaite, *Infinite families of links with trivial Jones polynomial*, preprint.

[S]   Kenneth Stephenson, *CIRCLEPACK, software for investigating circle packings*, available for download at http://www.math.utk.edu/∼kens.