MCS 521 – Combinatorial Optimization Fall 2013 Problem Set 2

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Due: 10/15/13 at the beginning of class

Related readings: Chapters 2 and 3

Instructions: Atop your problem set, please write your name and list your collaborators (see syllabus for the collaboration policy).

1. Consider the minimum spanning tree problem with a modified requirement – instead of minimizing the sum of the edge weights in the spanning tree, we want the weight of the maximum edge in the tree to be as small as possible. Does every MST also produce an optimal solution for this second requirement? Why or why not?

2. For the shortest paths problem, explain the relationship between Ford's algorithm and the Simplex method using the following statement: Let G be a connected digraph and $A = \{a_e : e \in E\}$ be its incidence matrix. A set $\{a_e : e \in T\}$ is a column basis of A if and only if T is the arc-set of a spanning tree of G. Prove the italicized statement above.

3. Using flows/cuts, prove that a bipartite graph in which every node has degree $k \ge 1$ has a perfect matching. How many disjoint perfect matchings must such a graph have?

4. Give an algorithm that finds a *legal ordering* of a graph in time $O(n^2)$.

5. Let G = (V, E) be a graph and $u_e \ge 0$ for all $e \in E$, and let $B \subseteq V$ and $A \subseteq V$. Show that

 $u(\delta(A)) + u(\delta(B)) \ge u(\delta(A \cup B)) + u(\delta(A \cap B)),$

i.e. that $u(\delta(.))$ is a submodular set function. This property is useful in establishing the Gomory-Hu structural condition.