MCS 521 – Combinatorial Optimization
Fall 2013
Problem Set 3

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Due: 11/26/13 at the beginning of class

Related readings: Relevant parts of chapters 4, 5, 7, and 8

Instructions: Atop your problem set, please write your name and list your collaborators (see syllabus for the collaboration policy).

1. Suppose G is a connected graph and has the property that for *every* node v there is a maximum matching of G not covering v. What are the strongest bounds you can prove on $\nu(G)$ as a function of $|V|$? Prove your answer correct. (*Hint: consider shrinking tight odd circuits.*)

2. Find the maximum matching and a minimizing set A in the Tutte-Berge formula for the graph in Figure 1 below.

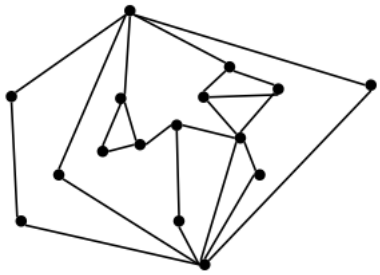

Figure 1: a graph

3. Give an example to show that the bound of $\frac{3}{2}\mathcal{OPT}$ for Christofides's heuristic cannot be improved. (*If you are unable to do so, give the tightest example you can.*) Explain why your example works.

4. Formulate the problem of finding a minimum-cost perfect matching in a bipartite graph as a minimum-cost flow problem.

5. Let $G = (V, E)$ be a graph and let

$$\mathcal{I} = \{J \subseteq E : \text{each component of the subgraph } (V, J) \text{ contains at most one circuit}\}.$$

Prove that $(E, \mathcal{I})$ is a matroid.