Math 215: Introduction to Advanced Mathematics
Problem Set 11

Due: Friday December 1

1) Let $n \in \mathbb{N}$. Suppose $A \subseteq \mathbb{N}_{2n}$ and $|A| = n + 1$. Prove that A contains a pair of distinct integers a, b such that a divides b. [**Hint**: Consider the function $f : A \to \{1, 3, 5, \ldots, 2n - 1\}$ where $f(a)$ = largest odd integer dividing a and apply the Pigeonhole Principle. Note that any $n \in \mathbb{N}$ can be written uniquely as $n = m2^M$ where m is odd.]

2) a) Suppose $f : X \to \mathbb{N}_m$ is injective but not surjective. Construct an injection $g : X \to \mathbb{N}_{m-1}$. Conclude that X is finite and $|X| \leq m - 1$.

b) Suppose Y is finite and $f : X \to Y$ is injective but not a surjection. Prove that $|X| < |Y|$.

c) Suppose X and Y are finite and $|X| = |Y|$, then every injection $f : X \to Y$ is a surjection.

3) a) Suppose $f : X \to Y$ is a surjection. We showed in Problem Set 8 that there is a right-inverse $g : Y \to X$ such that $f \circ g = I_Y$. Prove that g is injective.

b) Suppose X is finite and $f : X \to Y$ is surjective. Prove that Y is finite and $|Y| \leq |X|$.

c) Suppose X and Y are finite, $|X| = |Y|$ and $f : X \to Y$ is surjective. Prove that f is injective. [**Hint**: Apply 1c) to the right-inverse g to conclude that g is surjective and use this fact.]