MTHT 530 Analysis for Teachers II
Problem Set 4

Due: Wednesday February 8

1) a) Suppose $0 \leq f(x) \leq g(x)$ for all $x \in \mathbb{R}$ and $\lim_{x \to a} g(x) = 0$. Prove that $\lim_{x \to a} f(x) = 0$.
b) Suppose $g(x) \leq f(x) \leq h(x)$ for all $x \in \mathbb{R}$ and $\lim_{x \to a} g(x) = \lim_{x \to a} h(x)$. Prove that

$$\lim_{x \to a} g(x) = \lim_{x \to a} f(x).$$

[Hint: Use a)]

2) Using the definition of the derivative calculate $f'(2)$ for each of the following functions
a) $f(x) = \dfrac{1}{x^2}$
b) $f(x) = \sqrt{x}$.

3) Suppose $f, g, h : \mathbb{R} \to \mathbb{R}$ and $f(x) \leq g(x) \leq h(x)$ for all x, and $f'(a) = h'(a)$.
a) Suppose $f(a) = g(a) = h(a)$. Prove that g is differentiable at a and $g'(a) = f'(a) = h'(a)$.
b) Give an example showing this fails if we do not assume that $f(a) = g(a) = h(a)$.

4) a) Suppose $g : \mathbb{R} \to \mathbb{R}$ is continuous at 0. Let $f(x) = xg(x)$. Prove that f is differentiable at 0. What is $f'(0)$?

b) Suppose $f : \mathbb{R} \to \mathbb{R}$ is differentiable at 0 and $f(0) = 0$. Prove that there is function $g(x)$ continuous at 0 such that $f(x) = xg(x)$ for all $x \in \mathbb{R}$. What is $g(0)$?